MATH 2930, Fall 2018
TA: Aleksandra Niepla
Sections: 212, 217

Name:____________________________

Chapter 2.4, 2.5, 2.6 Review

Objectives: (1) Review theorems regarding existence and uniqueness for first order linear equations (2) Sketch phase lines and classify equilibrium solutions of autonomous differential equations (3) Solve exact differential equations

Part 1- Existence and Uniqueness Theorems

1. Consider a first order linear equation of the form

$$y' + p(t)y = g(t), \qquad y(t_0) = y_0.$$

Summarize the hypotheses needed to guarantee the existence and uniqueness of a solution to the above equation.

2. Do the same the following general first order non-linear equation:

$$y' = f(t, y), \qquad y(t_0) = y_0.$$

3. Without solving the problem, determine an interval in which a unique solution of the given initial value problem is certain to exist.

$$(4 - t^2)y' + 2ty = 3t^2, \qquad y(-3) = 1.$$

Part 2- Autonomous Differential Equations and Phase Line Sketches

1. An important class of first order equations consists of those in which the independent variable does not appear explicitly, i.e.

$$\frac{dy}{dt} = f(y).$$

Such equations are called ____________________________.

2. In the problem below, sketch the graph of $f(y)$ versus y, determine the equilibrium solutions (critical points), and classify each one as asymptotically stable or unstable.

$$\frac{dy}{dt} = y(y-1)(y-2), \qquad y_0 \geq 0.$$

Part 3- Exact Differential Equations

1. Let M, N, M_y, and N_x be continuous in the rectangular region R given by $\alpha < x < \beta, \gamma < y < \delta$. Then, the equation

$$M(x,y) + N(x,y)y' = 0$$

Is an **exact differential equation** in R if and only if ______________________________

at each point of R. That is, there exists a function ψ satisfying

$$\psi_x(x,y) = M(x,y) \qquad\qquad \psi_y(x,y) = N(x,y).$$

The solution is given implicitly as

$$\psi(x,y) = c.$$

2. Solve the given initial value problem

$$(2x - y) + (2y - x)y' = 0, \qquad y(1) = 3.$$