MATH 2930, Fall 2018
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Sections: 212, 217

Name:__________________________

Chapter 2.7, 3.1, 3.3 Review

Objectives: (1) Approximate solutions to first order differential equations using Euler's method (2) Practice solving second order linear homogeneous differential equations with constant coefficients

Part 1- Numerical Approximations: Euler's Method

1. Consider the initial value problem

$$y' = 0.5 - t + 2y, \qquad y(0) = 1$$

(i) Find approximate values of the solution of the given initial value problem at $t = 0.1, 0.2, 0.3$, and 0.4 using Euler's method with $h = 0.1$. Use the chart below:

n	t_n	y_n	$f(t_n, y_n)$	Line approximation at point (t_n, y_n) $y = y_n + f(t_n, y_n)(t - t_n)$
0	0	1		
1				
2				
3				
4				

(ii) Find the solution $y(t)$ of the given problem and evaluate $y(t)$ at $t = 0.1, 0.2, 0.3$, and 0.4. Compare your results with those of part (i).

Part 2- Second Order Linear Differential Equations

1. Find the general solution of the given differential equation

$$y'' + 2y' - 3y = 0.$$

2. Find the solution of the given initial value problem. Describe its behavior as t increases:

$$y'' + 8y' - 9y = 0 \qquad y(1) = 1, y'(1) = 0.$$

3. Find the general solution of the given differential equation

$$y'' + 6y' + 13y = 0.$$

4. Determine the values of α, if any, for which all solutions tend to zero as $t \to \infty$; also determine the values of α, if any, for which all nonzero solutions become unbounded as $t \to \infty$ for

$$y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0.$$

5. Consider the equation $ay'' + b'y + cy = 0$, where a, b, and c are constants with $a > 0$. Find the conditions on a, b, and c such that the roots of the characteristic equation are:
(i) real, different, and negative.
(ii) real with opposite signs.
(iii) real, different, and positive.
In each case, determine the behavior of the solutions as $t \to \infty$.

6. Solve the given equation for $t > 0$:

$$t^2 y'' + 4ty' + 2y = 0.$$

Hint: Let $x = \ln t$ and calculate dy/dt and d^2y/dt^2 in terms of dy/dx and d^2y/dx^2.