

Chapter 3.6, 3.7 Review

Objectives: (1) Using the method of variation of parameters to find a particular solution to second order linear differential equations (2) Talk about why second-order linear differential equations with constant coefficients are worth studying

Part 1: Variation of Parameters

Find the general solution of the following differential equation. When finding a particular solution first use the method of undetermined coefficients, and then use the method of variation of parameters.

1. $y'' - 5y' + 6y = 2e^t$

2. $4y'' - 4y' + y = 16e^{t/2}$

3. Find the general solution to the equation

$$y'' + y = \tan t.$$

Part 2: Mechanical and Electrical Vibrations

1. List some physical problems whose solutions are all described by solutions of the initial value problem:

$$ay'' + by' + cy = g(t), y(0) = 0, y'(0) = 0$$

2. Describe how the motion of a mass on a spring can be modeled by an equation of the form above.

3. A mass weighing 4 lb stretches a spring 2 in. Suppose that the mass is given an additional 6 in displacement in the positive direction and then released. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has a velocity of 3 ft/s. Formulate the initial value problem that governs the motion of the mass.

4. Assume that the system described by the differential equation $mu'' + \gamma u' + ku = 0$ is either critically damped or overdamped. Show that the mass can pass through the equilibrium position at most once, regardless of the initial conditions. (Hint: Determine all possible values of t for which $u = 0$).