

Chapter 5.4, 10.1-10.2 Review

Objectives: (1) To introduce and solve Euler Equations (2) To discuss and solve two-point boundary value problems (3) to introduce Fourier series and find the Fourier series of given functions

Part 1: Solving Euler Equations

Determine the general solution of the given differential equation that is valid in any interval not including the singular point.

1. $x^2y'' - 3xy' + 4y = 0$

2. $(x - 2)^2y'' + 5(x - 2)y' + 8y = 0$

3. $4x^2y'' + 8xy' + 17y = 0$

Part 2: Two-Point Boundary Value Problems

In each problem below, either solve the given boundary value problem or else show that it has no solutions.

1. $y'' + 2y = 0, \quad y'(0) = 1, y'(\pi) = 0.$

2. $x^2y'' + 5xy' + (4 + \pi^2)y = \ln x$, $y(1) = 0, y(e) = 0$.

3. Find the eigenvalues and eigenfunctions of the given boundary value problem. Assume that all eigenvalues are real,

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(L) = 0.$$

Part 3: Fourier Series

1. Sketch the graph of the function below for three periods. Then, find the Fourier series for the given function.

$$f(x) = \begin{cases} x + L, & -L \leq x \leq 0, \\ L, & 0 \leq x \leq L; \end{cases} \quad f(x + 2L) = f(x).$$