MATH 2930, Fall 2018
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Sections: 212, 217

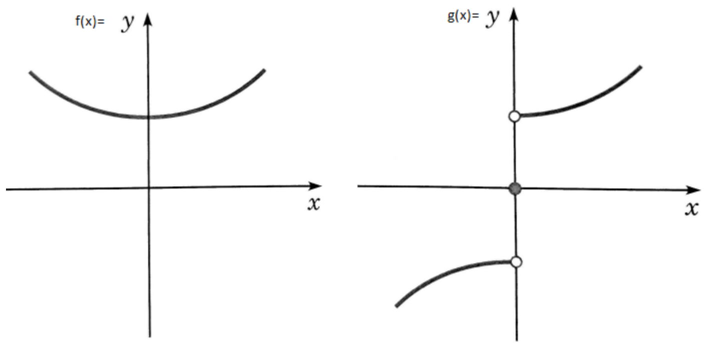
Name:____________________________

Chapters 10.4-10.6 Review

Objectives in class: (1) Review properties of odd and even functions (2) Practice the method of separation of variables to solve the heat equation with various boundary conditions. **Objectives at home:** (1) Study the lecture notes posted on Blackboard covering the derivation of the Heat Equation (2) carefully read Chapters 10.5-10.6.

Part 1: Odd and Even Functions

Consider the two functions graphed below:



(a) (b)

- Determine whether the following functions are odd, even, or neither:
  - $f(x) + g(x)$
  - $f(x)g(x)$
  - $f^2(x)$
  - $g^2(x)$
- Which function can be represented by a Fourier series of the form:

$$\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

- Which funciton can be represented by a Fourier series of the form:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

- Are the following functions odd, even, or neither
  - $\tan(2x)$
  - $\sec x$
  - e^{-x}

Part 2: Separation of Variables; Heat Equation

Consider a uniform bar of length L having an initial temperature distribution given by $f(x), 0 \leq x \leq L$. Write down the PDE modeling the following scenarios, and use the method of separation of variables to write down a the general solution for each.

1. Assume that the temperature of the bar is held fixed at zero at the end points.

2. Assume that the left end of the rod is held fixed at temperature T_1 and the right end is held fixed at temperature T_2.

3. Assume that the ends of the bar are insulated so that there is no passage of heat through them.

For each of the above, describe the steady state solution (i.e. $T(x) = \lim_{t\to} u(x,t)$*) Does your answer make sense with your intuition of the physical interpretation of the problem?*

Part 3: The Heat equation; concrete examples

1. Consider a rod of length $40cm$ made out of a material with thermal diffusivity constant equal to $\alpha = 10$. Suppose the end points are placed in an ice bath that keeps them at a constant temperature of 0 degrees Celsius. Find the temperature $u(x,t)$ of the rod if the initial temperature distribution is given by $\sin(2\pi x) + \sin(3\pi x)$ What is the steady state solution?

2. Repeat the above for the case that the right end is kept constant at a temperature of 30 degrees Celsius and the left end is kept constant at 10 degrees Celsius with the same initial temperature distribution of $\sin(2\pi x) + \frac{x}{2}$. What is the steady state solution?

3. Finally repeat the above for the case that the end points are kept insulated and the initial temperature distribution of $\cos(6\pi x)$. What is the steady state solution? Does this steady state solution make sense? Why?