Section Assignment #2 Solutions

Decibels are a unit of measure used to describe how loud a sound is. I_0 is the intensity of threshold sound, which is sound that can barely be perceived by the human ear. The loudness of a sound, in decibels, with intensity I is given by

$$dB = 10\log_{10}\left(\frac{I}{I_0}\right).$$

So, decibels are on a logarithmic scale of the intensity.

a) A cat's purr has and intensity that is about 316 times as intense as threshold sound. How many decibels is a typical cat's purr?

We are given the intensity of a cat's purr is $I = 316I_0$, (key words "times as intense as") so

$$dB = 10\log_{10}\left(\frac{316I_0}{I_0}\right) = 10\log_{10}(316) = 24.997.$$

That is, the cat's meow is about 24.997 decibels.

b) It is recommended that one wear ear protection when exposed to sounds louder than 85 decibels. Above what intensity should you be sure to wear ear protection if you plan to follow this guideline?

We are asked to find the intensity I which corresponds to 85 decibels. That is

$$85 = 10\log_{10}\left(\frac{I}{I_0}\right).$$

We'll use the fact that 10^x and $\log_{10} x$ are inverse functions.

$$\begin{aligned} 8.5 &= \log_{10}\left(\frac{I}{I_0}\right) \\ \implies 10^{8.5} &= \frac{I}{I_0} \\ \implies 10^{8.5}I_0 &= I. \end{aligned}$$

So, if the intensity of a sound is more than $10^{8.5}$ times the intensity of threshold sound, we should use ear protection.

c) A gunshot from a rifle has an intensity of about $2.5 \times 10^{13} I_0$. According to what you just figured out, do you think you should wear ear protection while firing a rifle?

Yes because $2.5 \times 10^{13} I_0 > 10^{8.5} I_0$. (This is just so you can compare with the intensity you calculated above, rather than compute the decibels to see if it is greater than 85.)

d) Suppose sound #1 has intensity $10I_0$ and sound #2 has intensity $100I_0$. Calculate and compare the loudness of each sound in decibels. What do you notice?

$$dB_1 = 10\log_{10}\left(\frac{10I_0}{I_0}\right) = 10\log_{10}(10) = 10(1) = 10.$$

$$dB_1 = 10\log_{10}\left(\frac{100I_0}{I_0}\right) = 10\log_{10^2}(10) = 20\log_{10} = 20.$$

Notice that even though the intensity of sound #2 is 10 times that of sound #1, the loudness in decibels is only twice that of sound #1. This is because decibels are on the logarithmic scale. It is convenient to use a logarithmic scale when dealing with large numbers. So for instance, here instead of using 10 & 100, it's like using 10 & 20.

The Richter scale is an even better example because it doesn't have the factor of 10 in front. For the Richter scale, the intensity of earthquakes are in terms of I_0 a threshold earthquake, which are movements that can barely be detected. The Richter rating of an earthquake of intensity I is give by

$$R = \log_{10}\left(\frac{I}{I_0}\right).$$

So for example, how much bigger is a 5.0 earthquake than a 4.0 earthquake? Well, solving for I as you did in part b above gives

$$I = 10^R I_0$$

So $R = 5$ vs $R = 4$ means $I = 10^5 I_0$ vs $I = 10^4 I_0$. So it is *ten times* as intense, even though the Richter rating only increased by 1. Again, this is because it is on a logarithmic scale. Basically, on a logarithmic scale we get to look at the exponents rather than the exponential function, which makes the numbers smaller and easier to deal with. However, it's important to remember when interpreting because one could think 4 to 5 doesn't sound like much of an increase, but it's really 10 times as strong.