Sampling, Confidence Intervals
Dr. Back

Oct. 26, 2009
Choose an SRS of size 5 from 30 people

**Step 1:** Assign ID numbers 01,02,..,29,30.  
all 2-digit!
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69 16 73 88 94 00 17 20 29 99 97 37 73 33 05

Our SRS is units 16 17 20 29 and 05.
Average Area?

Eyeball Estimate: Take a 5 second look and make a guess:
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Judgement Sample: Each person picks 5 representative rectangles, and computes their average area.
SRS: Each person uses a table of random numbers to choose 5 random rectangles, and computes their average area.
Stratified Random Sample: Call a rectangle **big** if its area is above 5; **small** otherwise.
Average Area?

**Stratified Random Sample:** Call a rectangle **big** if its area is above 5; **small** otherwise.

Have 70% of people find averages of big rectangles and 30% study small ones.
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\[0.48 \bar{x}_{\text{small}} + 0.52 \bar{x}_{\text{large}}\]
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\[ 0.48\bar{x}_{small} + 0.52\bar{x}_{large} \]

The numbers .48 and .52 come from the fact that there are 48 small and 52 big rectangles.
Average Area?

**Stratified Random Sample:** Call a rectangle **big** if its area is above 5; **small** otherwise.

Have 70% of people find averages of big rectangles and 30% study small ones.

The fractions 70% and 30% come from the following calculation:

<table>
<thead>
<tr>
<th>Type</th>
<th>$n_i$</th>
<th>$\sigma_i$</th>
<th>$n_i\sigma_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>small rectangles</td>
<td>48</td>
<td>1.557</td>
<td>74.74</td>
</tr>
<tr>
<td>big rectangles</td>
<td>52</td>
<td>3.847</td>
<td>200.04</td>
</tr>
</tbody>
</table>

\[
\frac{200.04}{200.04 + 74.74} = .73 \sim 70% \\
\frac{74.74}{200.04 + 74.74} = .27 \sim 30%
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These proportions give the stratified random sample with smallest variance for a given sample size.
Average Area?

Histogram of Actual Areas
Average Area?

Frequency Breakdown of Actual Areas

<table>
<thead>
<tr>
<th>Area</th>
<th>Frequency breakdown of No Selector</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
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<tr>
<td>2</td>
<td>2</td>
<td>18</td>
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<tr>
<td>3</td>
<td>6</td>
<td>24</td>
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<tr>
<td>4</td>
<td>15</td>
<td>39</td>
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<td>8</td>
<td>47</td>
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<tr>
<td>6</td>
<td>6</td>
<td>53</td>
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<td>8</td>
<td>8</td>
<td>61</td>
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<tr>
<td>9</td>
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<td>8</td>
<td>74</td>
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<tr>
<td>15</td>
<td>1</td>
<td>85</td>
</tr>
<tr>
<td>16</td>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td>18</td>
<td>5</td>
<td>100</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Group</th>
<th>Count</th>
<th>Cumulative</th>
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</tr>
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<tbody>
<tr>
<td>1</td>
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Total 100
Newspaper Poll

Poll size \( n = 400 \).
144 say yes, so

\[ \hat{p} = \frac{144}{400} = .36. \]
Newspaper Poll

Poll size $n = 400$. 144 say yes, so

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What does this say about the true proportion $p$?
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The sampling distribution of $\hat{p}$ is $N(p, SD(\hat{p}))$. 
Newspaper Poll

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What does this say about the true proportion \( p \)?

The sampling distribution of \( \hat{p} \) is \( N(p, SD(\hat{p})) \).

Recall \( SD(\hat{p}) = \sqrt{\frac{pq}{n}} \), \( SE(\hat{p}) = \sqrt{\frac{\hat{p} \hat{q}}{n}} \)
Newspaper Poll

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<th>( p )</th>
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<tr>
<td>.36</td>
<td>.0240</td>
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</tr>
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<td>.3</td>
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All these $\text{SD}(\hat{p})$ are quite close, so let’s approximate them all by $\text{SE}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .024$. 
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Our sampling distribution of \(\hat{p}\) is \(N(p, .024)\) approximately. For which \(p\) is \(\hat{p} = .36\) reasonably consistent with this?
CI Idea:

Answer will be an interval \((a, b)\) of possible values of \(p\).
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Use the sampling distribution of \(\hat{p}\) \((N(p, .24)\) in our example) (together with the confidence level) to decide which \(p\) are reasonably consistent.
CI Idea:

Include all values of the parameter $p$ which are reasonably consistent with the observed $\hat{p}$

Use the sampling distribution of $\hat{p}$ ($N(p, .24)$ in our example) (together with the confidence level) to decide which $p$ are reasonably consistent.

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At the level of the 68-95-99.7 rule, middle 95% means within 2 standard deviations.
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This shows $p = 0.5$ should not be in the CI:
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Picture when \( p \) is at the upper limit of the CI:
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Picture when $p$ is at the upper limit of the CI:

So the upper limit is $\hat{p} + .048 = .408$. 
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Picture when \( p \) is at the lower limit of the CI:
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Picture when $p$ is at the lower limit of the CI:

So the lower limit is $\hat{p} - 0.048 = 0.312$. 
CI Idea:

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A more accurate reading of Table Z tells us \( z = 1.96 \) is really the cutoff for the middle 95%.
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The value $z^* = 1.96$ is called a critical value.
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The same argument shows that our CI runs from $\hat{p} - z^*SE(\hat{p})$ to $\hat{p} + z^*SE(\hat{p})$
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Here

$$\hat{p} \pm z^*SE(\hat{p}) = 0.36 \pm (1.96)(0.024) = 0.36 \pm 0.047 = (0.313, 0.407)$$
Ci Idea:

e.g. for a 95% CI, a \( \hat{p} \) centered in the middle 95% zone of the sampling distribution of \( \hat{p} \) centered at \( p \) is reasonably consistent. We are 95% confident that the true value of \( p \) is between .313 and .407.