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Note Title 4/21/2016

$\mu \in \hat{T}$ dominant for $\Delta^+(\underline{k}, \underline{t})$

$\mu \longmapsto \lambda = \mu + 2\rho_c - \rho + \frac{1}{2}\nu \rightsquigarrow \underline{q} = \underline{l} + \underline{u}$

<u>COROLLARY</u> $\underline{l}_0$ is quasisplit and

$\check{\lambda}(\mu - 2\rho(u \cap p)) = 0$

<u>PROOF OF PROPOSITION</u>

G connected

$\Delta^+(\underline{k}, \underline{t})$ μ dominant

$\Delta^+(\underline{g}, \underline{t})$ $\mu + 2\rho_c$ dominant

Fix a $\delta \in \hat{K}$ & suppose

$$\left[H^J(u \cap k, \mu') \otimes \Lambda^{J''}(u \cap p)^* \right]^{\mu - 2\rho(u \cap p)} \neq 0$$

for $(J, J'') \neq (0, R)$

AIM FOR $\|\mu'\|_\lambda < \|\mu\|_\lambda$

- ϕ weight of H^J $\phi = \sigma(\mu' + \rho_c) - \rho_c$
- ψ weight of $\Lambda^{J''}$

$\mu - 2\rho(u \cap p) = \phi - \psi$

Let $J' = R - J$ $\quad (J, J') \neq (0, 0)$

$\exists \{\gamma_i\} \in \Delta(u \cap p)$

$\psi = 2\rho(u \cap p) - \sum_{J'} \gamma_i$

$\sigma(\mu' + \rho_c) - \rho_c = \mu - \sum_{i=1} \gamma_i$

$\mu' + 2\rho_c \longleftrightarrow (\Delta^+)' \longmapsto \lambda'$

$\lambda = \mu + 2\rho_c - \rho + \frac{1}{2} v$

$\lambda' = \mu' + 2\rho_c - \rho' + \frac{1}{2} v'$

$\rho_n = \rho(\Delta^+(p, t)), \ \rho_n' = \rho(\Delta^{+\prime}(p, t))$

$\sigma^{-1}\lambda = \lambda' + (\rho_n' - \frac{1}{2} v') - \sigma^{-1}(\rho_n - \sum \gamma_i - v)$

AIM FOR $\langle \lambda', \lambda' \rangle < \langle \lambda, \lambda \rangle$.

<u>LEMMA</u> $\{Q = \rho_n - \sum c_\beta \beta\}$
with $0 \le c_\beta \le 1$ is independent of Δ^+ & W_K-invariant

$\exists \ \{a_\gamma : \gamma \in \Delta^{+\prime}\} \ 0 \le a_\gamma \le 1$

$\sigma^{-1}(\rho_n - \sum \gamma_i - \frac{1}{2} v) = \rho_n' - \sum a_\gamma \gamma$

SO

$\sigma^{-1}\lambda = \lambda' + \sum a_\gamma \gamma - \frac{1}{2} v'$

$\langle \lambda, \lambda \rangle = \langle \sigma^{-1}\lambda, \sigma^{-1}\lambda \rangle \geq \langle \lambda', \lambda' \rangle$

because

$\langle \lambda', \sum a_\gamma \gamma - \nu' \rangle = \langle \lambda', \sum a_\gamma \gamma \rangle \geq 0$

"=" $\iff \sum a_\gamma \gamma - \nu' = 0$

If "=" $\sigma^{-1}\lambda = \lambda'$ & both are $\Delta^+(\underline{k}, t)$-dominant, so $\sigma \in W_K^1$

$\Rightarrow \sigma = 1$, $\lambda = \lambda'$ & Δ^+ Δ'^+ differ only by roots $\perp \lambda$.

$\lambda = \lambda - \sum \gamma_i + (\text{terms} \perp \lambda)$

$\langle \lambda, \lambda \rangle = \langle \lambda, \lambda \rangle - \langle \lambda, \sum \gamma_i \rangle$

& $\gamma_i \in \Delta(u \cap p)$, so no γ_i.

$J = \ell(\sigma) = 0$, $J' = |\{\gamma_i\}| = 0$

PROOF OF THE LEMMA

$A \subseteq \Delta(p, t)$ any set. $\ni$

1) $|A| = \frac{1}{2} |\Delta(p, t)|$

2) $A \cup -A = \Delta(p, t)$

Put $S(A, e) := \sum e_\beta \beta \quad \beta \in A, -\frac{1}{2} \leq e_\beta \leq \frac{1}{2}$

Claim These are the weights in the lemma.

Fix $Q = \rho_n - \sum c_\beta \beta$

$$A = \{\beta \in \Delta^+(\mathfrak{p}, t) : c_\beta \leq \tfrac{1}{2}\} \cup \{-\beta : \beta \in \Delta^+ : c_\beta > \tfrac{1}{2}\}$$

$$e_\beta = \begin{cases} \frac{1}{2} - c_\beta & \beta \in \Delta^+ \\ c_\beta - \frac{1}{2} & \beta \notin \Delta^+ \end{cases}$$

The $S(A, e) \longleftrightarrow Q$'s