SPECTRAL SEQUENCE

Note Title 4/19/2016

$$A \xrightarrow{d} A \qquad d^2 = 0$$

$$A = A^0 \supset A^1 \supset \cdots \supset A^n \supset \cdots$$

a filtration compatible with d

$$d(A^p) \subset A^p$$

$$H(A) := \ker d / \operatorname{im} d$$

$$A \rightsquigarrow gr(A) := \bigoplus gr^p(A) = \bigoplus A^p/A^{p+1}$$

$$D^p = (A^p \cap \ker d)/(A^p \cap \operatorname{im} d)$$

gives a filtration of $H(A)$

WANT $gr\,H(A)$, next best thing to $H(A)$.

$$E^p_\infty = D^p/D^{p+1}$$

CONSTRUCT

$$Z^p_r = \{z \in A^p : dz \in A^{p+r}\} \quad r \geq 0$$

$$Z^p_{-1} = A^p$$

$$d Z^{p-r}_r \subseteq Z^p_r \subseteq A^p, \quad Z^p_{r+1} \subset Z^p_r, \quad Z^{p+1}_{r-1} \subseteq Z^p_r$$

$$E^p_r := Z^p_r / (d Z^{p-r+1}_{r-1} + Z^{p+1}_{r-1})$$

$$E^p_0 = A^p/A^{p+1} \qquad E^p_1 = H(\bar{d}, A^p/A^{p+1})$$

$d_r^p : E_r^p \longrightarrow E_r^{p+r}$ $\quad H(d_r^p) = E_{r+1}^p$

ASSUME $A = \bigoplus {}^nA$, $d : {}^nA \to {}^{n+1}A$ compatible with A^p.

$Z_r^{p,q} := {}^{p+q}A \cap Z_r^p$ etc.

FIRST QUADRANT $A^0 = A$, $A^{p,q} = 0$ for $q < 0$

THEN • $E_r^{p,q} = E_{r+1}^{p,q} = E_\infty^{p,q}$ for $r > \max(p, q+1)$

• $\bigcap D^p = (0)$