MATH 1340, HOMEWORK #6

DUE THURSDAY, MARCH 23

Please *show your work* in the problems that require calculations. For the short answer questions, write in complete sentences. All assertions must be justified to get full credit.

0. (For participation credit.) Answer the response question on Piazza. This time, please submit your own response before responding to a classmate's.

1. [Exam #1 2(a)] A social choice procedure is said to satisfy the **criterion of unanimity** if an alternative is the *unique* social choice, whenever every ballot has that alternative ranked highest. Prove that if a social choice procedure satisfies the Pareto criterion, then it must also satisfy the criterion of unanimity.

2. [Exam #1, 2(b)] A social choice procedure is said to satisfy the **criterion of non-imposition** if every alternative occurs as the unique winner for at least one set of ballots. (In other words, for every candidate in the running, there exists some scenario of votes in which that candidate can be chosen.) Prove that if a social choice procedure satisfies unanimity, then it must also satisfy the criterion of non-imposition.

3. [Exam #1 Bonus: Copeland's method and "semi-Condorcet" candidates] Suppose that the number of voters is odd. We say that a candidate is **semi-Condorcet** if it wins at least 50% of its head-to-head matches with other candidates. Show that if a candidate x is a chosen as a winner under Copeland's method (i.e. has the best win-loss record), then x is semi-Condorcet.

4. Show that the Banzhaf and Shapley indices are always identical when there are only two voters. (One way to do this is to analyze all possible collections of winning coalitions in a voting system with only two voters, and check in all cases that the Banzhaf indices and Shapley-Shubik indices coincide.)

5. [TP 3.40]

6. [TP 3.41]