Math 1340:
Mathematics & Politics

Lecture 1, January 26, 2017

Course Information

- Instructor: Brian Hwang
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  Office Hours: Tuesdays, 2:45-3:45pm, or by appointment
- TA: Gautam Gopal Krishnan
- Office Hours: TBA

More Course Info

- Textbook (required): *Mathematics and Politics: Strategy, Voting, Power, and Proof* by Alan Taylor and Allison Pacelli. (an electronic copy is free for Cornell students)
- Website (for homework, handouts, slides): www.math.cornell.edu/~bhwang/1340/

Evaluation

- Homework (40%) will be assigned each Thursday and will be due one week later.
- Exam I (25%): March 16
- Exam II (25%): Date/Time TBA
- Participation/Responses (10%): On Piazza, see HW#1 for details.

Course Outline

- Voting and Social Choice
  - How do we choose a winning candidate in an election?
  - Voting methods ("social choice functions")
  - Two candidates? Multiple candidates?
  - What properties should a voting method have? Does there exist a voting method that has all of the desired characteristics?
  - How should we vote to pass bills or amendments?

- Political Power
  - Who do you need to ally with to have the vote go in your favor?
  - How do we measure political power?
  - Who has most power (according to these measures): the House, the Senate, or the President?

- Conflict and Game Theory
  - How do you model real-world conflicts as simple games?
  - How do you maximize your payoff in a competitive game, if your opponent is trying to do the same?
  - Zero-sum games, strategies, Nash equilibria, theory of moves (sequential games)

- Fairness (Apportionment, Fair Division)
  - How should the votes of the House be distributed?
  - How should you divide resources to maximize everybody's happiness? (e.g. dividing a cake, mining rights, property in divorce)

- Escalation (Auction theory)
  - How should auctions be run?
  - How should you bid in a different kinds of auctions?
  - What kinds of information should change the way you bid?

Math & Politics?!

Voting and Social Choice

Electing a class president

- Suppose we must elect a class president and that there are two candidates.
- We need to choose a voting method (a.k.a. social choice function) which selects a candidate (or candidates) given the votes of the class.
- For simplicity, assume that everyone casts a vote by writing the name of exactly one of the candidates on their ballot.

What method should we use to pick the winner?

- Any thoughts?

Voting and Social Choice

- What this section is NOT about:
  - who the candidates are, their policies, their actions, speeches, debates, etc.
  - who the voters are, registration, voter suppression, suffrage, etc.
  - how votes are cast, voting machines, etc.
- What is it about: once the votes are cast, how do we determine who wins?

On Definitions

- Throughout this course, we will need to be precise about our definitions. (This is still a math course.)
- Often, a definition of a term may clash with its common usage. We will take the ones introduced in class as authoritative.

Social Choice Functions

- A function is a rule that assigns to every possible input from one set (called the domain) a single output in another set (called the codomain).
- Functions cannot be indifferent: they must *always* output something for a given input in the domain.
- Functions cannot change their minds: if a function outputs "orange" on input "Brian," it must *always* output "orange" on input "Brian."

Definition: Social Choice Function

- A **social choice function** for *two* candidates A and B is a function whose domain is the set of all possible voting results for a given electorate and whose codomain is the set consisting of "A wins," "B wins," and "tie."

(Non-)examples

- Suppose we pick a candidate by a coin flip: heads, A wins; tails, B wins. Is this a social choice function?
- How about the method that picks the candidate with the most votes?

- The social choice function matters! Let's see an example of how three different "reasonable" voting systems work in a real-life example:
  - traditional American method
  - instant-runoff procedure
  - pairwise comparison ("head-to-head")