MATH 6670, HOMEWORK #8

DUE THURSDAY, MARCH 28TERM

1. (Getting acquainted with QC sheaves.) This is essentially a walkthrough (and generalization) of the example given in class.

(a) Let A be a commutative ring, $S \subset A$ a multiplicative set, and M_i an infinite collection of A-modules, indexed by $i \in I$. Show that, in general, the map

$$\left(\prod_{i\in I} M_i\right)_S \to \prod_{i\in I}(M_i)_S$$

is not an isomorphism.

(b) Give an example of how an infinite product of quasi-coherent sheaves is not quasi-coherent.

(c) Let X_i be a scheme for all i in some index set I, and let $X = \sqcup_{i\in I} X_i$ be the disjoint union of all such schemes. Let $\mathscr{F}_i$ be a sheaf of abelian groups on X_i, thought of as a sheaf on X by being zero on all X_j for $j \neq i$. Show that in this case, the map

$$\bigoplus_{i\in I} \mathscr{F}_i \to \prod_{i\in I} \mathscr{F}_i$$

is an isomorphism.

(d) Using part (b) and (c), produce an example of a (non-quasicompact) map $f : X \to Y$ and a quasicoherent sheaf $\mathscr{F}$ on X such that $f_*(\mathscr{F})$ is not quasi-coherent. (Hint: What if X is the disjoint union of infinitely many copies of Y?)

2. [H II.2.16(a)–(d)] [This is an exercise that slowly goes through a proof technique we've been using a lot to prove things in class.]

3. [H II.3.11](a)–(d)] (Closed subschemes)

4. [H II.3.16] (Noetherian induction) [This is another example of these local-to-global kinds of results that we saw in class.] and [H II.3.19] Give a proof of Chevalley's theorem following the outline in [H II.3.19].

5. [H II.5.17] (Affine morphisms)