1. (a) $z - 6 = 4(x-1) + 4(y-2)$
   (b) $L(x,y) = 6 + 4(x-1) + 4(y-2)$, so

$$\begin{aligned} L(1.01, 1.9) &= 6 + 4(0.01) + 4(-0.1) \\ &= 6 + 0.04 - 0.4 = 5.64. \end{aligned}$$

   (c) $\mathbf{L}(t) = \mathbf{P} + t\mathbf{V}$, where $\mathbf{P} = (3,5,9)$ and $\mathbf{V}$ is perpendicular to the plane in (a). Rewriting the plane equation as $4x + 4y - z = 6$, we see that one possibility is just $\mathbf{V} = (4,4,-1)$ (the normal vector of the plane).

2. (a) The level set is $x^2 + y^2 + z^2 = 4$, i.e. a sphere in $\mathbb{R}^3$ centered at the origin with radius 2.
   (b) $\nabla f(\mathbf{X}) = -(2/r^4)\mathbf{X}$, where $\mathbf{X} = (x,y,z)$ is a point and $r^2 = x^2 + y^2 + z^2$.

   So at $\mathbf{P} = (0,0,2)$, the greatest rate of change is $\|\nabla f(\mathbf{P})\| = (2/r^4)\|\mathbf{P}\| = (2/16)(2) = 1/4$.

   The direction of greatest change is $\mathbf{V} = \nabla f(\mathbf{P})/\|\nabla f(\mathbf{P})\| = (0,0,-1)$.
   (c) The directional derivative is $D_{\mathbf{V}}(\mathbf{P}) = \nabla f(\mathbf{P}) \cdot \mathbf{V} = (-2/r^4)(2/\sqrt{2}) = (-1/8)(2/\sqrt{2}) = -\frac{1}{4\sqrt{2}}$.

   (At $(0,0,2)$, moving in a direction with positive z (for eg. our current $\mathbf{V}$) is a direction that brings $\mathbf{P}$ further away from the origin, which decreases the value of f.)

3. (a) Let $\mathbf{U} = \mathbf{X} - \mathbf{Y}$ and $\mathbf{V} = \mathbf{Y}$. The triangle inequality applied to $\mathbf{U}$ and $\mathbf{V}$ gives

$$\begin{aligned} \|\mathbf{U} + \mathbf{V}\| &\leq \|\mathbf{U}\| + \|\mathbf{V}\| \\ \|\mathbf{X} - \mathbf{Y} + \mathbf{Y}\| &\leq \|\mathbf{X} - \mathbf{Y}\| + \|\mathbf{Y}\| \\ \|\mathbf{X}\| - \|\mathbf{Y}\| &\leq \|\mathbf{X} - \mathbf{Y}\|. \end{aligned}$$

   Then let $\mathbf{U} = \mathbf{X}$ and $\mathbf{V} = \mathbf{Y} - \mathbf{X}$. The triangle inequality applied to $\mathbf{U}$ and $\mathbf{V}$ gives

$$\begin{aligned} \|\mathbf{U} + \mathbf{V}\| &\leq \|\mathbf{U}\| + \|\mathbf{V}\| \\ \|\mathbf{X} + \mathbf{Y} - \mathbf{X}\| &\leq \|\mathbf{X}\| + \|\mathbf{Y} - \mathbf{X}\| \\ \|\mathbf{Y}\| - \|\mathbf{X}\| &\leq \|\mathbf{Y} - \mathbf{X}\|. \end{aligned}$$

   The two above inequalities can be combined using the absolute value sign to give the answer.
   (b) Let $\varepsilon > 0$ be given. For $X, Y \in \mathbb{R}^n$ we have

$$\begin{aligned} \|f(X) - f(Y)\| &= |\ k\|X\| - k\|Y\|\ | \\ &= |k|\ |\ \|X\| - \|Y\|\ | \\ &\leq |k|\ \|X - Y\|, \end{aligned}$$

   where we use part (a) for the inequality.

   If $k = 0$, then any $\delta > 0$ will give the result. Otherwise choose $\delta = \varepsilon/|k|$. Then for any X, Y so that $\|X - Y\| < \delta$, then $\|f(X) - f(Y)\| < |k|\varepsilon/|k| = \varepsilon$, proving uniform continuity.

4. Since $f : \mathbb{R}^n \to \mathbb{R}$ is linear, there exists a row vector A^T so that $f(X) = A^T X = A \cdot X$.
   Then $|f(X)| = |A \cdot X| \leq ||A||\ ||X||$ by Cauchy-Schwarz. Pick $c = \|A\|$.

5. $DF = \begin{bmatrix} \cos v & -u \sin v & 0 \\ \sin v & u \cos v & 0 \\ 0 & 0 & 1 \end{bmatrix}$, with $\det DF = u$. Thus, the inverse function theorem guarantees a local inverse for F everywhere except along $u = 0$, i.e. $\{x = 0, y = 0\}$ (the z axis).

6. (a) From $f(x, y, z(x, y)) = 0$ we take the partial wrt x of both sides to obtain

$$f_x + f_z z_x = 0.$$

(b) Repeat part (a) for $y(x, z)$ (partial wrt z) and $x(y, z)$ (partial wrt y) to obtain

$$\begin{aligned} f_y y_z + f_z &= 0 \\ f_x x_y + f_y &= 0. \end{aligned}$$

Rearranging these three equations, we have

$$\begin{aligned} z_x &= -\frac{f_x}{f_z} \\ y_z &= -\frac{f_z}{f_y} \\ x_y &= -\frac{f_y}{f_x}. \end{aligned}$$

Multiplying these three equations together, we find $x_y y_z z_x = -1$.