

MATH 2220 PRELIMINARY EXAM 2
MARCH 24TH, 2015

Name_____

1. Let $F(x, y, z) = (x - 1)^2 + y^2 + z^2/4 - 1$ and let

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid F(x, y, z) = 0\}.$$

Find all the points $P \in S$ such that the tangent plane to S at P is parallel to the plane

$$x + y + z = 1. \quad (15\text{pts})$$

2. Let $f(x, y) = \sin(x + y)$.

- a) Estimate $f(0.1, 0.1)$ using linear approximation. (5pts)
- b) Show that the error term in the linear approximation above is less than 0.02. (10pts)
- c) Can you show that the error term in the linear approximation given above is actually less than 0.002? (5pts) **Omit 2c**

3. Let

$$D = \{(x, y) \mid x^2 + (y - 1)^2 \leq 4\}$$

and let $f : D \longrightarrow \mathbb{R}$ be given by

$$f(x, y) = 3x - x^3 + y^2 - 2y.$$

Find the maximum and minimum of f on D . (20pts)

4. Let

$$F_1(x, y, u, v) = e^x y^2 + v \sin(x + u)$$

and

$$F_2(x, y, u, v) = x + y + v \cos(x + u).$$

Show that the equations $F_1 = 0 = F_2$ can be used to find u and v in terms of x and y in a neighborhood of $(-1, 0, 1, 1)$. (15pts)

5. Let $\mathbf{c}(t) = (t, t^2, 2t^3/3)$ for $t \in [0, 1]$. Compute the arc length of the curve defined by the path $\mathbf{c}$. (15pts)

6. Let D be the region bounded by the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$. Compute

$$\iint_D \sqrt{1-x^2} dA. \quad (15\text{pts})$$