YOUR NAME ________________________________

MATH 2220 PRELIM 2 April 5, 2016

This is a 90 minute test. No notes or calculators are allowed. There are 6 questions. Please write your answers on the lined paper provided. Be sure to write your name and netid on each sheet of paper you use for your answers. Show all your work. 'Answers only' rarely earn full credit.

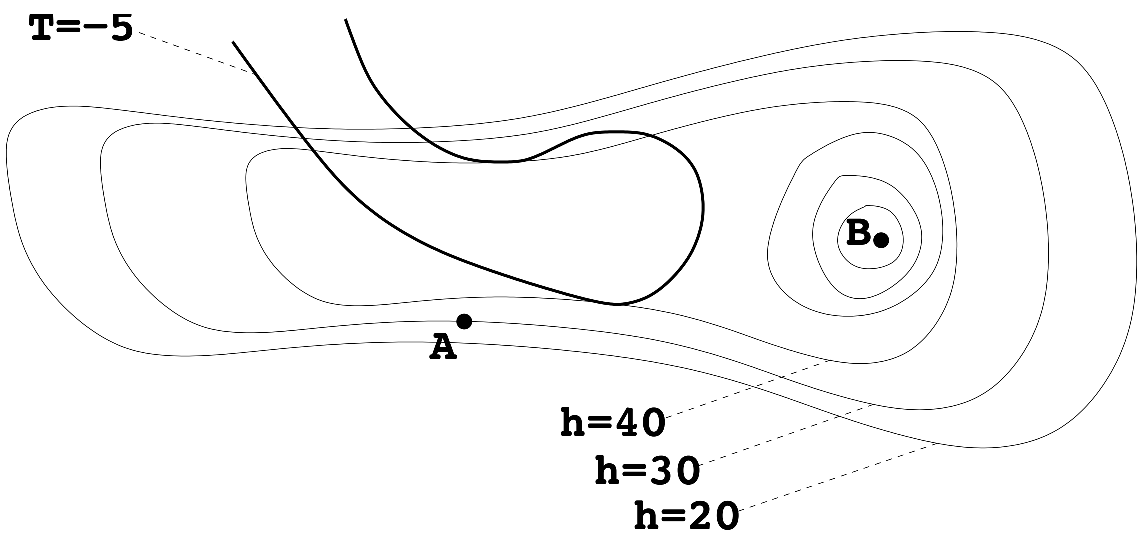
1. Let $p(x, y) = (1/\pi)e^{-(x^2+y^2)}$.

   (a) Show that p is a probability density function. Show all your work.

   (b) Find the probability that $0 \leq x$, $0 \leq y$, and $x \leq y$.

2. The figure below shows some level curves of a function h that gives the elevation $h(x, y)$ at the point (x, y). It also shows one level set $T = -5$ of a temperature function T. Throughout the xy plane T and h have continuous first and second partial derivatives and $\nabla T \neq \vec{0}$.

(a) Indicate the direction of the gradient vector, $\vec{\nabla} h(A)$, as accurately as you can at the point A in the figure.



(b) Mark any points on the level sets $h = 20, h = 30, h = 40$ that the Lagrange Multiplier method would identify as possible places where h may have a maximum or minimum when $T = -5$. (Be sure to explain that the points you selected satisfy the Lagrange multiplier method equations.)

(c) Suppose h has a local maximum or a local minimum at B. What is $\vec{\nabla} h(B)$? What condition on the Hessian matrix of h at$B$ would assure that $h(B)$ is a local maximum?

3. The transformation $x = au + bv$, $y = cu + dv$ maps the circular disk of radius 1 centered at the origin in the u, v plane, one-to-one and onto a region R in the x, y plane.

   (a) Find the area of R.

   (b) Is the transformation orientation preserving, orientation reversing, or is it not possible to tell from the given information?

4. Let D be the region in described in spherical coordinates as being bounded by the surfaces given by $\rho = 2$, $\rho = 1$, $\phi = \pi/6$ and $\phi = \pi/2$.

   (a) Sketch the region D.

   (b) Suppose the density at the point (ρ, ϕ, θ) is $\rho \cot \phi$. Use spherical coordinates to find the mass of D. (Note: $\cot \phi = \frac{\cos \phi}{\sin \phi}$)

5. Suppose that g is continuous on R^2 and that $0 \leq g(x, y)) \leq (1/\pi)e^{-(x^2+y^2)}$.

(a) Prove that g is integrable on R^2. (Hints: To show that g is integrable on R^2 be sure to first state what it means for g to be integrable over the unbounded set R^2. Then verify that g satisfies those conditions. By Problem 1 on this test you can conclude that $f(x, y) = (1/\pi)e^{-(x^2+y^2)}$ is integrable over R^2.)

(b) Find a constant c so that cg is a probability density function.