MATH 2220 PRELIM 1 September 29, 2015

YOUR NAME _____

This is a 90 minute test. No notes or calculators are allowed. There are 6 questions. Please write your answers in the space provided after the question. Show all your work. 'Answers only' rarely earn full credit.

1. Let $f(x, y) = e^{x-y} + \log_e y$.

   (a) Find an equation of the plane tangent to the graph of f at the point where $x = 1$ and $y = 1$.

   (b) Find the absolute value of the volumetric flow rate (i.e flux) of the constant vector field $\mathbf{F} = (2, 3, 1)$ across a parallelogram determined by the vectors $(0, 1, f_y(1, 1))$ and $(1, 0, f_x(1, 1))$.

   Ignore 1(b)

3. Let $\mathbf{F}$ be a linear function from R^n to R^m. Prove that $\mathbf{F}$ is uniformly continuous on R^n.

4. The figure below shows level curves of a C^1 function f from R^2 to R and a unit vector u. Determine the signs (positive, negative, or zero) of the directional derivatives of f in the direction of U at P, Q, and R. Justify your answers using the dot product.
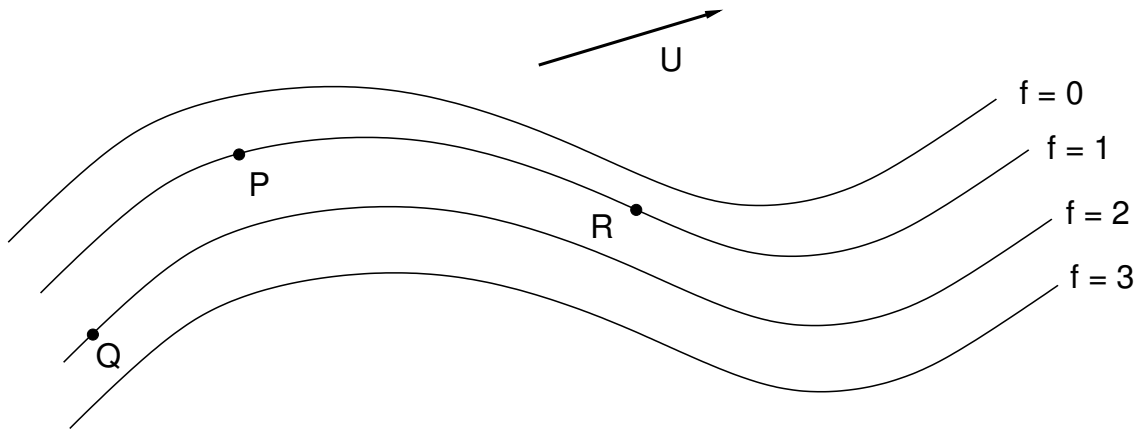
5. Let $g(x, y) = f(u(x, y), v(x, y))$. Suppose f, u, and v are C^2 functions and $u(x, y) = y$.

   (a) Show that $g_x = f_v v_x$ and $g_y = f_u + f_v v_y$.

   (b) Find g_{xx}.

6. Show that near the point $(x_1, x_2, y_1, y_2) = (2, 3, \pi, 1)$ the system

$$x_1 y_2 - x_1 \cos y_1 = 5$$

$$x_2 \sin y_1 + x_1 y_2 = 2$$

can be solved for y_1 and y_2 in terms of x_1 and x_2 and find the partial derivatives of y_1 and y_2 with respect to x_1 at $(2, 3)$.