YOUR NAME ____________________

MATH 2220 PRELIM 1 February 25, 2016

This is a 90 minute test. No notes or calculators are allowed. There are 6 questions. Please write your answers on the lined paper provided. Be sure to write your name and netid on each sheet of paper you use for your answers. Show all your work. 'Answers only' rarely earn full credit.

1. Let $f(x, y) = 2x^2 + y^2$.

   (a) Find an equation of the plane tangent to the graph of of f at the point where $x = 1$ and $y = 2$.

   (b) Estimate the value of $f(1.01, 1.9)$ using a linear approximation to f at $(1, 2)$.

   (c) Find an equation for the line that passes through the point $(3, 5, 9)$ and that is perpendicular to the plane you found in part a).

2. Let $f(x,y,z) = \frac{1}{x^2+y^2+z^2}$.

(a) Sketch the c level set of f where $c = \frac{1}{4}$.

(b) Find the greatest rate of change in f at $(0,0,2)$. At the point $(0,0,2)$ in which direction is the the rate of change in f the greatest? Sketch that direction on your figure from part a).

(c) Is the rate of change in f at $(0,0,2)$ in the direction $(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}})$ positive, negative or zero? Give a reason for your answer.

3. Let X, Y be any vectors in R^n.

(a) Show that $\Big| ||X|| - ||Y|| \Big| \leq ||X - Y||$.

(b) Show that for any real number k. the function $f(X) = k||X||$ is uniformly continuous on R^n.

4. Suppose f is a linear function from R^n to R. Show that there is a non negative number c so that for any X in R^n, $|f(X)| \leq c||X||$.

5. Let $F(u, v, w) = (u \cos v, u \sin v, w)$ and let $x = u \cos v$, $y = u \sin v$, and $z = w$. At which points in R^3 does the Inverse Functions Theorem guarantee a local inverse for F exists? Show how you are using the theorem to justify your answer.

6. Suppose f is a C^1 function from R^3 to R.

(a) Suppose $f(x, y, z) = 0$ and that z is a differentiable function of (x, y). *Use the Chain Rule* to derive an expression relating z_x, f_x, and f_z.

(b) Show that if $f(x, y, z) = 0$ and if each of the variables , x, y and z is a differentiable function of the other two variables, i.e, $x(y, z)$, $y(x, z)$, and $z(x, y)$ then

$$x_y y_z z_x = -1.$$