Math 661 – Geometric Topology (homework 2, due Sep 20)

Exercise 2.1. Let ζ^1 be a 1-cycle in the plane and $\mathcal{T}$ a finite triangulation of the convex hull of the graph supporting ζ^1. Moreover assume that the graph supporting ζ^1 is contained in the 1-skeleton of $\mathcal{T}$, i.e., the edges of the triangles in $\mathcal{T}$ cover the support of ζ^1. For each triangle $\Delta \in \mathcal{T}$, the number

$$\mathrm{rot}_{\zeta^1}(\Delta)$$

is defined as the rotation of ζ^1 around an interior point of Δ. We have seen that this is well defined. Show that the 2-chain

$$\zeta^2 := \sum_{\Delta \in \mathcal{T}} \mathrm{rot}_{\zeta^1}(\Delta)\, \Delta$$

bounds ζ^1.

Exercise 2.2. Find two open subsets U and V of the sphere $\mathbb{S}^2$ such that the map

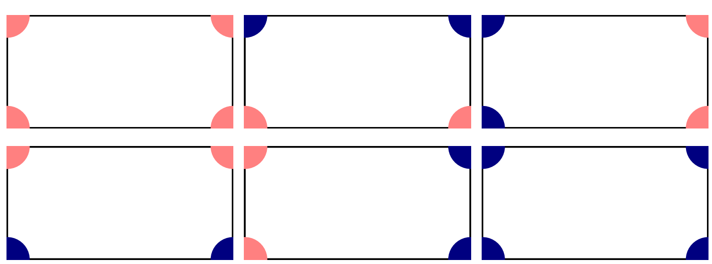
$$\mathrm{H}_1(U \cap V) \to \mathrm{H}_1(U) \oplus \mathrm{H}_1(V)$$

is *not* injective.

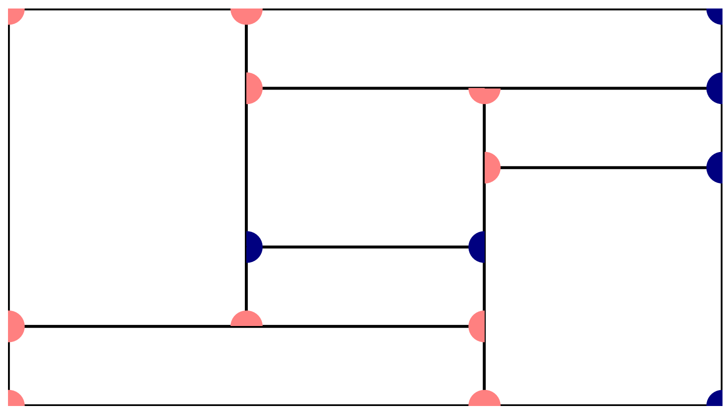
Exercise 2.3. Let $\iota : \mathbb{E}^2 \to \mathbb{S}^2$ denote the stereographic projection. Show that for each open subset $U \subseteq \mathbb{E}^2$,

$$\mathrm{H}_1^{\mathbb{E}^2}(U) = \mathrm{H}_1^{\mathbb{S}^2}(\iota(U))\,.$$

Exercise 2.4. Consider a rectangle that is tiled by subrectangles. Each of the subrectangles has its corners coloured by red and blue according to one of the following six patterns:



Prove that the induced colouring of the four corners of the big rectangle also fits one of these patterns provided that in the interior the colours around vertices that belong to different rectangles match. Here is an example:



Exercise 2.5. Let C be a compact set in the plane $\mathbb{E}^2$. Suppose the complement has precisely two components and each point in C is *arcwise accessible* from both components. (A point in C is arcwise accessible from a given complementary component if it is the endpoint of an arc that is, away from this endpoint, completely contained in a complementary component.) Show that any two point subset of C disconnects C.

Exercise 2.6. Let $f : \mathbb{S}^1 \to \mathbb{E}^2$ be continous and injective. Show that f is a homeomorphism from $\mathbb{S}^1$ onto its image.

Each problem is worth 5 points, but you can earn at most 20 point with this assignment.

Late homework will not be accepted.