Math 661 – Geometric Topology (homework 4, due Oct 04)

Exercise 4.1. Show that every arc in a surface Σ is bicollared, i.e., every embedding $f : \mathbb{I} \hookrightarrow \Sigma$ of the unit interval extends to an embedding $\tilde{f} : \mathbb{I} \times [-1, 1] \hookrightarrow \Sigma$ such that $\tilde{f}(t, 0) = f(t)$. (The case where Σ is an open set in the plane $\mathbb{E}^2$ is worth 4 points of partial credit. The plane itself is worth 3 points.)

Exercise 4.2. Let A be an embedded arc in the surface Σ with endpoints P and Q. Let A' be another embedded arc in Σ. Show that A can be moved by an "ambient deformation" so that almost all intersections of the two arcs disappear. More precisely, show that there is a homeomorphism $\varphi : \Sigma \to \Sigma$ such that

1. The points P and Q are fixed by φ.
2. The homeomorphism is isotopic to the identity relative $\{P, Q\}$.
3. The set $\varphi(A) \cap A'$ is finite.

(The case where Σ is an open set in the plane $\mathbb{E}^2$ is worth 4 points of partial credit. The plane itself is worth 3 points.)

Exercise 4.3. Let Γ be a finite graph embedded in the sphere $\mathbb{S}^2$. Show that there is a homeomorphism $\varphi : \mathbb{S}^2 \to \mathbb{S}^2$ such that the edges of $\varphi(\Gamma)$, which also is an embedded graph, are broken geodesics.

Exercise 4.4. Let L_0 and L_1 be subdivisions of the simplicial complex K. Show that L_0 and L_1 are combinatorially equivalent, i.e., they have isomorphic subdivisions.

Exercise 4.5. Let $\mathcal{T}$ be a triangulation of the surface Σ. Show that $\mathcal{T}$ has a subdivision all of whose 2-simplices are Jordan domains, i.e., closed discs in Σ that are contained in the domain of a chart.

Exercise 4.6. Let $\mathcal{T}$ and $\mathcal{T}'$ be two triangulations of the surface Σ. Suppose that any edges in $\mathcal{T}$ intersects any edge in $\mathcal{T}'$ in at most finitely many points. (I am suppressing the homeomorphisms $|\mathcal{T}| \to \Sigma$ and $|\mathcal{T}'| \to \Sigma$ in the wording.) Show that $\mathcal{T}$ and $\mathcal{T}'$ are combinatorially equivalent, i.e., they have subdivisions that are isomorphic as simplicial complexes.

Each problem is worth 5 points, but you can earn at most 20 points with this assignment.

Late homework will not be accepted.