Math 661 – Geometric Topology (homework 9, due Nov 09)

Exercise 9.1. Show that the map

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} : z \mapsto \frac{-1}{z}$$

is an orientation preserving isometry of $\mathbb{H}^2$.

Exercise 9.2. Show that, in the upper half plane model, the map

$$z \mapsto \frac{1}{\overline{z}}$$

is a reflection along a geodesic.

Exercise 9.3. Show that hyperbolic circles in the upper half plane model are Euclidean circles. (The Euclidean and hyperbolic center, however, do not coincide.)

Exercise 9.4. In a hyperbolic triangle, we define angles by drawing Euclidean tangent lines and measuring their Euclidean angles. Show that the area of a triangle with angles α, β, and γ is $\pi - \alpha - \beta - \gamma$.

Exercise 9.5. Show that a metric space is proper if an only if:

$$\text{compact} \iff \text{closed and bounded}$$

Exercise 9.6. Show that a geodesic metric space is proper if it is complete and locally compact.

Each problem is worth 5 points, but you can earn at most 20 points with this assignment.

Late homework will not be accepted.