Math 739 – Important Groups (homework 8, due Mar 29)

Exercise 8.1. Prove that the map

$$x_i : t \mapsto \begin{cases} t & \text{for } 0 \leq t \leq i \\ i + \frac{1}{2}(t-i) & \text{for } i \leq t \leq i+2 \\ t-1 & \text{for } i+2 \leq t \end{cases}$$

defines an action of F on $[0, \infty]$.

Exercise 8.2. Prove that the words with even exponent sum for the generator x_0 form a subgroup of index 2 in F which is isomorphic to F.

Exercise 8.3. Show that the centralizer of x_1 in F is isomorphic to $F \times C_\infty$.

Exercise 8.4. Show that

$$\langle \ldots x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots \mid x_m x_i = x_i x_{m+1} \text{ for } i < m \rangle$$

embeds into F as a normal subgroup with infinite cyclic quotient. Note that this group is not finitely generated.

Definition. The Baumslag-Solitar group is defined by the presentation

$$BS(1,2) := \left\langle x, y \,\middle|\, x^y = x^2 \right\rangle$$

Exercise 8.5. Show that the presentation complex for $BS(1,2)$ is contractible and one-ended.

Exercise 8.6. Prove that $BS(1,2)$ has exponential growth.

Exercise 8.7. As $BS(1,2)$ is solvable, we know that it is amenable and has a Følner sequence. Prove that $BS(1,2)$ has no Følner sequence consisting of balls in the Cayley graph defined by the presentation.

It suffices to solve, on the average, half of the problems correctly.