Math 739 – Important Groups (homework 13, due May 3)

Exercise 13.1. Show that the alternating group A_{11} is a quotient of $\mathrm{SL}_2(\mathbb{Z})$.

Exercise 13.2. Prove that for every $m \in \mathbb{N}$, the homomorphism

$$\mathrm{SL}_2(\mathbb{Z}) \to \mathrm{PSL}_2(\mathbb{Z}_m)$$

is onto.

Exercise 13.3. Fix a prime p and a natural number k. Show that

$$N_{p,k} := \left\{ \begin{pmatrix} 1+xp^k & yp^k \\ zp^k & 1-xp^k \end{pmatrix} \mod p^{k+1} \,\middle|\, 0 \leq x, y, z < p \right\} \leq \mathrm{SL}_2\left(\mathbb{Z}_{p^{k+1}}\right)$$

is a normal abelian p-subgroup of rank 3 in $\mathrm{SL}_2\left(\mathbb{Z}_{p^{k+1}}\right)$ such that

$$\mathrm{SL}_2\left(\mathbb{Z}_{p^{k+1}}\right)/N_{p,k} \cong \mathrm{SL}_2\left(\mathbb{Z}_{p^k}\right).$$

Exercise 13.4 (extra credit). Show that SQ-universal groups have uncountably many non-isomorphic quotients.

It suffices to solve, on the average, half of the problems correctly.