Math 758 – *Your* Favorite Groups (homework 4, due Feb 28)

Exercise 4.1. Prove that a CAT(1) piecewise spherical simplicial complex is metrically flag.

Exercise 4.2. Show that every properly proper action is proper, and that every proper action on a proper metric space is properly proper.

Exercise 4.3. Let G act properly and cocompactly on the length space X. Show that X is complete and locally compact.

Let $G = \langle \Sigma \rangle$ be a finitely generated group. For every element $g \in G$, the translation length (with respect to Σ) is defined as

$$\underline{\tau}(g) := \lim_{k \to \infty} \frac{|g^k|}{k}.$$

Exercise 4.4. Show that the translation length of a group element is well defined, i.e., the limit exists and is independend of the generating set Σ.

Exercise 4.5. Let G be virtually free abelian of rank n, and let H be a subgroup of G that is free abelian of rank n. Show that H has finite index in G. (Hint: Consider a finite index $\mathbb{Z}^n$ inside G and the action of H on $G/\mathbb{Z}^n$.)

Exercise 4.6. Let G be finitely generated. Show that G is virtually abelian provided the commutator subgroup $[G, G]$ is finite. (Hint: Let H be the centralizer of $[G, G]$ in G and show that the center of H has finite index in H and that H has finite index in G.)

Each problem is worth 5 points, but you can earn at most 20 points with this assignment.

Late homework will not be accepted.