1. **Homework: Due Tuesday, September 18**

Exercise 1.6, Exercise 1.7, Exercise 1.9, Problem 1.8, Problem 1.9, Problem 1.10, Problem 1.11.

2. **Homework: Due Thursday, September 27**

Solve the following problem:

**Problem 2.1.** Let $f$ be a $2\pi$-periodic function which is piecewise smooth on $\mathbb{R}$. Show that $S_N(f)(\theta)$ converges to $\frac{f(\theta+) + f(\theta-)}{2}$ for every $\theta$ on the torus. Then, use this fact to show that $\zeta(2) = \frac{\pi^2}{6}$ and that $\zeta(4) = \frac{\pi^4}{90}$.

(Hint: To calculate the zeta values, consider functions of type $\theta$, $\theta^2$, $\theta^3$, $\theta^4$ and extend them periodically to the whole real line.)

3. **Homework: Due Thursday, November 1**

Exercise 2.11, Problem 2.12, Problem 3.9, Problem 3.10 and the following:

**Problem 3.1.** Prove the following “triangle inequality” for the quasi-Banach space $L^{1,\infty}(\mathbb{R})$:

$$\|f_1 + \ldots + f_N\|_{1,\infty} \leq C \log N (\|f_1\|_{1,\infty} + \ldots + \|f_N\|_{1,\infty}).$$

Then show that the inequality is sharp in the sense that the $\log N$ factor cannot be removed or replaced by a smaller one (such as $(\log N)^{1/2}$, for example).

(Hint: For the inequality, show that one can reduce the problem to the case where all the functions involved take values between $1/N$ and 1 on their supports. Then, show that if $f$ is such a function in $L^{1,\infty}(\mathbb{R})$ so that $1/N < f(x) \leq 1$ for every $x$ in the support of $f$, then one has $\|f\|_1 \leq C(\log N)\|f\|_{1,\infty}$. For the counterexample, consider the function $\sum_{j=1}^{N} \frac{1}{j} \chi_{[j,j+1]}$ and its rearrangements.)