Math 294 Homework Assignment, Wednesday Nov 8, 2000

1. Consider the differential equation

$$my'' + cy' + ky = 0 \qquad ()' = d/dt$$

which is the differential equation that models the motion of the mass m when it is connected to a spring with stiffness k and also to a dashpot with viscosity c. (That is, the spring exerts force $-ky$ on the mass when it is displaced distance y and the dashpot exerts force $-cy'$ on the mass when it travels at velocity y'.) This equation can be solved by reducing it to a system of two first order differential equations in terms of the variables $x_1 = y$ and $x_2 = y'$ and then we can write the equation in the form

$$\mathbf{x}' = A\mathbf{x}$$

where x is the column vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and A is a 2×2 matrix.

i. Assuming $m = 1, c = 6$ and $k = 4$ set up the system and find the general solution first in terms of $\mathbf{x}$ and then in terms of y.

ii. If the initial conditions are $y = 1, y' = 0$, give the solution in terms of y and in terms of x_1 and x_2.

iii. Let $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be a new set of variables that can be written as $\mathbf{z} = B\mathbf{x}$ for a suitable 2×2 matrix B. Find B such that the system can be written as $\mathbf{z}' = D\mathbf{z}$ where D is a diagonal matrix. For the initial conditions given in ii) write out the solution in terms of z_1 and z_2.

iv. Solve the problem again but this time assume $m = 1, c = 2, k = 4$.

2. Consider the differential equation

$$y''' + 2y'' - y' - 2y = 0$$

This equation can be solved by reducing it to a system of three first order differential equations in terms of the variables $x_1 = y, x_2 = y'$ and $x_3 = y''$ and then can write the equation in the form

$$\mathbf{x}' = A\mathbf{x}$$

where $\mathbf{x}$ is the column vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and A is a 3×3 matrix.

v. Set up the system and find the general solution first in terms of $\mathbf{x}$ and then in terms of y. (Hint: One of the eigenvalues is -2.)

vi. If the initial conditions are $y = 1$, $y' = 0$ and $y'' = 0$, give the solution in terms of y and in terms of x_1, x_2, and x_3.

vii. Let $\mathbf{z} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a new set of variables that can be written as $\mathbf{z} = B\mathbf{x}$ for a suitable 3×3 matrix B. Find B such that the system can be written as $\mathbf{z}' = D\mathbf{z}$ where D is a diagonal matrix. For the initial conditions given in ii) write out the solution in terms of z_1, z_2, and z_3.