Math 762 Homework Assignment, Due Thursday, March 1

1. Let $G(p)$ be the tensegrity where the configuration p is the set of vertices of a cube in $\mathbb{E}^3$, struts connect antipodal vertices and cables connect all the other pairs of vertices. Let Ω_1 be the stress matrix correponding to having a proper stress of 1 on the edges of the cube, -1 on the struts, and 0 on the other members. Let Ω_2 be the stress matrix correponding to having a stress of 1 on the face diagonals, -2 on the struts, and 0 on the rest. Show that each of these stresses are equilibrium stresses. The matrix Ω_1 is positive definite and Ω_2 is indefinite with a negative eigenvalue. So for some $0 < t < 1$, $(1-t)\Omega_1 + t\Omega_2$ will have a 5-dimensional kernel. In other words, the universal configuration for that stress will have at least an affine span that is 4-dimensional. Find that critical value of t, and describe the corresponding universal configuration.

2. Let $p = (p_1, p_2, p_3, p_4, p_5, p_6)$ be a configuration of points in the plane that form a regular hexagon in order. Let $G(p)$ be the tensegrity where the the edges of the hexagon are bars and the three antipodal pairs of vertices are cables. Show that there is a continuous "flex" of this configuration that is not a congruence. (Hint: You can even fix the length of one of the cables to make it a bar.)

3. Let $G(p)$ be the tensegrity where the configuration p is the set of vertices of a convex planar polygon plus one further vertex in its interior, the edges of the polygon are cables and the interior vertex is connected to each of the polygonal vertices by a strut. Show that there is a non-zero proper equilibrium stress for $G(p)$ whose stress matrix Ω has kernel of dimension 3 and exactly one negative eigenvalue. (Hint: Use induction starting with the case that the polygon is a triangle.) You may assume that the interior vertex does not lie on any line connecting vertices of the polygon, to make things easier, if you wish.