

Annotated references for rigidity/packing/granular materials  
Session on Granular Matter  
Institut Henri Poincaré  
R. Connelly  
Cornell University  
Department of Mathematics

The first place to look for a given reference, given the author's name and approximate time of publication is in Math Reviews on line at MathSciNet (<http://www.ams.org/mathscinet/search>) or Zentralblatt (<http://www.emis.de/ZMATH/>). Both of these give you some free searches, but it is more convenient to work through a library work station or use, say, your institution's access, if it exists. Often, but not always, you can link from those sites to the journal's archive. Some places that have archived several journals is Jstor (<http://www.jstor.org/>) and a particularly difficult journal to find in libraries, which has a lot of articles on rigidity from the mathematical perspective, is Structural Topology. It is now no longer publishing, but all issues are available at <http://www-iri.upc.es/people/ros/StructuralTopology/>. Another place to look for preprints is the archive at <http://front.math.ucdavis.edu/>. Of course, it is often useful to google the author and hunt in his/her web page.

The following are some of my papers from MathSciNet.

MR2061244 (2005a:74011) Donev, Aleksandar; Torquato, Salvatore; Stillinger, Frank H.; Connelly, Robert A linear programming algorithm to test for jamming in hard-sphere packings. *J. Comput. Phys.* 197 (2004), no. 1, 139--166. 74A99 (52C17 65K05 90C05)

This explains how linear programming is used to do numerical simulations and free log jams of circle or spherical packings.

MR1861203 (2002g:82022) Connelly, Robert; Rybnikov, Konstantin; Volkov, Stanislav Percolation of the loss of tension in an infinite triangular lattice. *J. Statist. Phys.* 105 (2001), no. 1-2, 143--171. (Reviewer: Raphaël Cerf) 82B43

This explains the lack of a positive percolation threshold for rigidity that needs tension for stability. This is in my lecture V.

MR1626699 (99c:52027) Bezdek, A.; Bezdek, K.; Connelly, R. Finite and uniform stability of sphere packings. *Discrete Comput. Geom.* 20 (1998), no. 1, 111--130. (Reviewer: S. Stein) 52C17 (52C25)

This discusses the rational for a sense of rigidity for large packings, other than just infinitesimal rigidity. Also the collapsing square lattice is described.

MR1402190 (97e:52037) Connelly, Robert; Whiteley, Walter Second-order rigidity and prestress stability for tensegrity frameworks. *SIAM J. Discrete Math.* 9 (1996), no. 3, 453--491. (Reviewer: Tiong Seng Tay) 52C25 (70B15)

This is where we define and prestress stability, second-order rigidity, and show how they are connected. The theory here is used to prove some conjectures of B. Roth about cabled polygons in the plane. This is the main reference for Lecture VI here.

MR1254089 (94m:52027) Connelly, Robert; Servatius, Herman Higher-order rigidity--- what is the proper definition? *Discrete Comput. Geom.* 11 (1994), no. 2, 193--200. (Reviewer: Tiong Seng Tay) 52C25

This is an example that shows that a naïve definition of higher-order rigidity does not have reasonable properties.

MR1102004 (92i:52023) Connelly, Robert Juxtapositions rigides de cercles et de sphères. II. Juxtapositions infinies de mouvement fini. [Rigid circle and sphere packings. II. Infinite packings with finite motion] Dual French-English text. *Structural Topology* No. 16 (1990), 57--76. (Reviewer: A. Florian) 52C15 (05B40 52C25)

This explores some of the alternate definitions of stability for infinite packings. The theory here is applied to several symmetric packings to determine their stability by the various definitions. This is relevant to Lectures III and IV.

MR1242981 (94j:52041) Connelly, Robert Rigidity. *Handbook of convex geometry*, Vol. A, B, 223--271, North-Holland, Amsterdam, 1993. (Reviewer: Tiong Seng Tay) 52C25 (70C20 73K99)

This is a survey of some of the basic results in mathematical rigidity as it is relevant to convexity.

MR0939609 (89i:52022) Connelly, Robert Juxtapositions rigides de cercles et de sphères. I. Juxtapositions finies. [Rigid circle and sphere packings. I. Finite packings] Dual French-English text. *Structural Topology* No. 14 (1988), 43--60. (Reviewer: A. Florian) 52A45

This explains how the basic rigidity theory applies to packings of circles and spheres. The canonical push is presented showing the importance of infinitesimal rigidity.

MR0652643 (83m:52012) Connelly, Robert Rigidity and energy. *Invent. Math.* 66 (1982), no. 1, 11--33. (Reviewer: B. G. Roth) 52A25 (51M20 73K99)

This is starting point for stress component of the Hessian of a more-or-less arbitrary energy functional. This can be used to show the stability of a wide variety of tensegrity structures used by artists, for example.

MR0591730 (82a:53059) Connelly, Robert The rigidity of certain cabled frameworks and the second-order rigidity of arbitrarily triangulated convex surfaces. *Adv. in Math.* 37 (1980), no. 3, 272--299. (Reviewer: B. G. Roth) 53C45 (52A25)

The title is the theorem. Actually, arbitrarily triangulated convex polyhedra in 3-space, even with judiciously placed holes in the faces, are prestress stable, which is a bit stronger statement.

MR0551682 (80k:53089) Connelly, Robert The rigidity of polyhedral surfaces. *Math. Mag.* 52 (1979), no. 5, 275--283. (Reviewer: H. R. Gluck) 53C45 (51N05 57M20 73K05)

This is the best write-up of my example of a flexible triangulated surface that is

embedded in 3-space.

Some of my papers not in Math Reviews:

Donev, I. Cisse, D. Sachs, E. Variano, F. Stillinger, R. Connelly, S. Torquato, P.M. Chaikin, Improving the Density of Jammed Disordered Packings Using Ellipsoids, *SCIENCE*, 13 February, 2004, Vol. 303 ([www.sciencemag.org](http://www.sciencemag.org)).

The remarkable thing here is that in general ellipsoid packings are not infinitesimally rigid, and they tend to pack more densely than packings of spheres.

"Jamming in Hard Sphere and Disk Packings", by A. Donev, S. Torquato, F. H. Stillinger, and R. Connelly, *J. Appl. Phys.*, Vol. 95 (3): 989, 2004

Numerical computations and an argument for the concept of strictly jammed packings.

"Comment on "Jamming at zero temperature and zero applied stress: The epitome of disorder", by A. Donev, S. Torquato, F. H. Stillinger, and R. Connelly, *Phys. Rev. E*, Vol. 70: 043301, 2004

Comments on related papers and correcting misconceptions about our work.

Papers by others:

MR1730205 Whiteley, Walter Rigidity and scene analysis. *Handbook of discrete and computational geometry*, 893--916, CRC Press Ser. Discrete Math. Appl., CRC, Boca Raton, FL, 1997. 65D18 (05C99)

This is a survey of some of the general literature on rigidity from wide perspective.

MR1237630 (94g:52028) Crapo, Henry; Whiteley, Walter Autocontraintes planes et polyèdres projetés. I. Le motif de base. (French) [Plane self stresses and projected polyhedra. I. The basic pattern] Dual French-English text. *Structural Topology* No. 20 (1993), 55--78. (Reviewer: Robert Connelly) 52C25 (52B70 73H99 73K05)

This is a discussion of the Maxwell-Cremona correspondence and other matters related to stresses on frameworks.

MR1165538 (93h:52029) Whiteley, Walter Matroids and rigid structures. *Matroid applications*, 1--53, *Encyclopedia Math. Appl.*, 40, Cambridge Univ. Press, Cambridge, 1992. (Reviewer: Robert Connelly) 52C25 (05B35 52B40 65Y25 68U05 70C20)

This is a survey of several of the results related to rigidity from the combinatorial point of view and matroids.

MR0804977 (87e:05139) Tay, Tiong-Seng; Whiteley, Walter Generating isostatic frameworks. Dual French-English text. *Structural Topology* No. 11 (1985), 21--69. (Reviewer: G. Laman) 05C99 (51K99 52A37)

Basic constructions using Hennenberg moves to create isostatic bar frameworks.

MR0610958 (82m:51018) Roth, B.; Whiteley, W. Tensegrity frameworks. Trans. Amer. Math. Soc. 265 (1981), no. 2, 419--446. (Reviewer: Robert Connelly) 51F99 (52A37 53A17 73K20)

The basic duality result as discussed in Lecture II.

MR0531609 (81j:73066a) Bolker, Ethan D.; Crapo, Henry Bracing rectangular frameworks. I. SIAM J. Appl. Math. 36 (1979), no. 3, 473--490. (Reviewer: Colin J. H. McDiarmid) 73K99 (05B35)

This is the beginning article that discusses the homework problem.

``Unusually Dense Crystal Packings of Ellipsoids'', by A. Donev, F. H. Stillinger, P. M. Chaikin and S. Torquato, Phys. Rev. Lett., 92, 255506, 2004

The title is the result.

S. Torquato, T. M. Truskett and P. G. Debenedetti, ``Is Random Close Packing of Spheres Well Defined?'' Physical Review Letters, 84, 2064 (2000).

This discusses the difficulty of describing what is happening when uniform spherical balls are packed "randomly" in a container.

See <http://cherrypit.princeton.edu/sal.html> and <http://atom.princeton.edu/donev/> for more references about the activities of Sal Torquato's group at Princeton.

MR1481894 (98g:65143)

Jacobs, Donald J.(1-MIS-PA); Hendrickson, Bruce(1-SAND-AN)

An algorithm for two-dimensional rigidity percolation: the pebble game. (English. English summary)

J. Comput. Phys. 137 (1997), no. 2, 346--365.

65Y25 (82C99)

This is an introduction to the pebble game to determine the generic rigidity of a bar framework in the plane.

MR0400239 (53 #4074) Gluck, Herman Almost all simply connected closed surfaces are rigid. Geometric topology (Proc. Conf., Park City, Utah, 1974), pp. 225--239. Lecture Notes in Math., Vol. 438, Springer, Berlin, 1975. (Reviewer: Robert Connelly) 57C05 (53C45 57A05)

This is a good description of the infinitesimal rigidity of convex triangulated, Dehn's Theorem. But his proof has a gap for the case when the faces are not triangles.