Research Statement

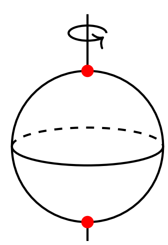
David Mehrle
dfm223@cornell.edu
math.cornell.edu/~dmehrle

My research is in *equivariant stable homotopy theory*, which is the study of stable invariants of spaces equipped with a group action. In this field, we replace homotopy groups by *homotopy Mackey functors*, which are an algebraic gadget that records a system of topological invariants and relations between them. Although computations in equivariant stable homotopy theory are done with Mackey functors, many aspects of their algebra have not yet been developed. The goal of my thesis is to understand homological and commutative algebra in the category of Mackey functors. Work of mine with Mike Hill and J.D. Quigley finds a surprising difference between free algebras in the category of Mackey functors and ordinary free algebras (Section 2). In another project with J.D. Quigley, I investigate an equivariant Hochschild–Kostant–Rosenberg theorem, which could lead to new calculations in equivariant or Hermitian K-theory (Section 3).

1 Background: Equivariant homotopy theory and Mackey functors

Equivariant homotopy theory is the study of algebraic invariants of spaces or spectra equipped with a group action. It has been instrumental in several recent advances in algebraic topology, such as Hill–Hopkins–Ravenel's solution to the Kervaire invariant one problem [HHR16] and calculations of algebraic K-theory via trace methods [BoHM93, HM03, NS18]. Here, we will consider only finite groups G.

Invariants of G-spaces should distinguish between different group actions on the same space. For example, the cyclic group of order three can act on a 2-sphere in two different ways. Either it does nothing, or it rotates the sphere like a globe through an angle of 120°. The homotopy groups of the sphere can't tell the difference between the different actions, but the homotopy groups of the fixed points can. The fixed points of the trivial action are the whole sphere, while the fixed points of rotation are the north and south poles.



The cyclic group C_3 acts on S^2 by rotation with fixed points S^0 (in red).

Here's some intuition for why the fixed points show up. Non-equivariantly, picking a point in a space X can be done by choosing a map from a point into X. There is a homeomorphism between the mapping space of all such maps and the space itself:

$$\mathrm{Map}(\{*\}, X) \cong X.$$

Equivariantly, whenever you pick a point in a G-space X, you must say how the group acts on that point. Instead of specifying a point in X, this amounts to specifying an entire G-orbit. We say that "orbits are the points of equivariant homotopy theory." Any G-orbit in X is homeomorphic to a coset space G/H for some subgroup $H \subseteq G$, and can be picked out by a G-equivariant map from the coset space G/H to our space X. The mapping space of all such G-equivariant maps is homeomorphic to the H-fixed points of X:

$$\mathrm{Map}^G(G/H, X) \cong X^H. \tag{1.1}$$

Since orbits are the basic building blocks of equivariant homotopy theory, we should take care to keep track of the morphisms between them. To do so, we form the G-*orbit category* $\mathbf{Orb}_G$, whose objects are the coset spaces G/H and whose morphisms are all G-equivariant maps between them. The collection of fixed point spaces for a G-space X can be encoded in the functor

$$\mathrm{Map}^G(-, X)\colon\ \mathbf{Orb}_G^{op} \to \mathbf{Top}.$$

On objects, this functor takes the orbit G/H to the H-fixed points of the space X by (1.1). A morphism $G/H \to G/K$ in $\mathbf{Orb}_G$ becomes an inclusion of fixed points $X^H \hookrightarrow X^K$. In fact, all of the homotopy theory of G-spaces is contained in such functors: Elmendorf's Theorem [Elm83] says that the category of G-spaces and the category of contravariant functors from the orbit category to topological spaces have the same homotopy theory.

Post-composing one of these functors $\mathrm{Map}^G(-, X)$ with a homotopy group functor π_n gives us the correct invariant for equivariant homotopy theory, called a *homotopy coefficient system* of X. It consists of the homotopy groups of all of the fixed point spaces of X, as well as the homomorphisms induced on homotopy groups by the inclusions of fixed point spaces. Generically, a *coefficient system* is a contravariant functor from the orbit category to abelian groups. The name comes from the fact that coefficient systems are the coefficients for equivariant cohomology theories.

Another example of a coefficient system is the fixed points of a G-module M. We will call this coefficient system $\underline{M}$; as a functor, $\underline{M}$ takes an orbit G/H to the H-fixed points of M and takes morphisms of $\mathbf{Orb}_G$ to inclusions of fixed points. There is more structure here than just the data of a coefficient system. From the additive structure of M, we get homomorphisms in the other direction to the inclusions. For subgroups $K \subseteq H \subseteq G$, a K-fixed point $m \in M^K$ becomes an H-fixed point by summing over the action by the K-cosets of H:

$$\begin{aligned} M^K &\longrightarrow M^H \\ m &\longmapsto \sum_{gK \in H/K} g \cdot m. \end{aligned} \tag{1.2}$$

This is our first example of a *Mackey functor*, called the *fixed point functor* of the G-module M.

A *Mackey functor* [Dre73] is a pair of functors $\underline{M} = (\underline{M}_*, \underline{M}^*)$ subject to the following conditions. Both $\underline{M}_*$ and $\underline{M}^*$ are functors from the orbit category to abelian groups, but $\underline{M}_*$ is covariant and $\underline{M}^*$ is contravariant. These two functors must agree on objects; we write $\underline{M}(U)$ for the common value $\underline{M}^*(U) = \underline{M}_*(U)$. This must satisfy a formula analogous to the double coset formula. In a Mackey functor, the contravariant morphisms $\underline{M}^*(f)$ are called *restrictions*, and the covariant morphisms $\underline{M}_*(g)$ are called *transfers*. A morphism of Mackey functors from $\underline{M}$ to $\underline{N}$ is a natural transformation that works for both the covariant and contravariant functors simultaneously. We write $\mathbf{Mack}_G$ for the category of G-Mackey functors.

Examples of Mackey functors are abundant. Group cohomology, homology, and Tate cohomology can be expressed as Mackey functors. The representation ring of any finite group is a Mackey functor in which transfers are induction and restriction is restriction of representations to a subgroup. The earlier example of the fixed points of any G-module is the Mackey functor for the zeroth group cohomology.

The relevant examples for my research are the homotopy Mackey functors of a G-spectrum. For instance, a G-space has stable homotopy Mackey functors, much as a space has stable homotopy groups.

Because Mackey functors play the role of abelian groups in equivariant stable homotopy theory, we tend to think of them as algebraic objects instead of functors. This is borne out in the properties of the category of G-Mackey functors. The category $\mathbf{Mack}_G$ is abelian, so it makes sense to do homological algebra and to talk about free, projective, and flat Mackey functors. There is a symmetric monoidal product on $\mathbf{Mack}_G$, called the *box product*. When G is the trivial group, the category of G-Mackey functors is equivalent to the category of abelian groups, and the box product is the tensor product over $\mathbb{Z}$. In this sense, the category of Mackey functors generalizes the category of abelian groups.

Commutative monoids for the box product are called *Green functors*. Green functors are in many ways analogous to commutative rings. Loosely speaking, a Mackey functor $\underline{M}$ is a Green functor when $\underline{M}(U)$ is a commutative ring for all finite G-sets U, suitably compatible with the other data of $\underline{M}$. If R is a commutative ring with an action of G by ring homomorphisms, then the fixed point Mackey functor of R is a Green functor. Write $\underline{R}$ for this fixed point Green functor.

This fixed point functor has more structure. In the fixed point Mackey functor for a G-module M, we obtained transfer homomorphisms (1.2) from the additive structure of the G-module. In the fixed point Green functor $\underline{R}$, there is also a morphism that comes from the multiplicative structure. For subgroups $K \subseteq H \subseteq G$, a K-fixed point $r \in R^K$ becomes an H-fixed point by multiplying over the action by K-cosets of H:

$$\begin{aligned} R^K &\longrightarrow R^H \\ r &\longmapsto \prod_{gK \in H/K} g \cdot r. \end{aligned}$$

We call this these the *norm morphisms*. They are extra structure that is not present in the definition of Green functors as commutative monoids for the box product.

Just as cohomology rings are more powerful invariants than cohomology groups, the extra structure of norms makes analysis easier. A Green functor $\underline{R}$ together with norm morphisms $nm_K^H\colon \underline{R}(G/K) \to \underline{R}(G/H)$ compatible with the rest of the data of $\underline{R}$ is called a *Tambara functor* [Tam93]. A Tambara functor is another kind of commutative ring in the category of Mackey functors. Tambara functors are the more honestly equivariant object, insofar as they are the monoids for an equivariant symmetric monoidal structure on the category of Mackey functors [HM19].

Sometimes we don't have all of the norm morphisms, but only some of them[1]. It is useful to remember which ones we do have. A Green functor with some, but not all, of the norms is called an *incomplete Tambara functor* [BH18]. The norms that are present in an incomplete Tambara functor are described by an *indexing system* [BH15]. There is a finite lattice of indexing systems ordered by which norms are present. The greatest element of this lattice is the indexing system corresponding to Tambara functors, with all norms. The least element is the indexing system corresponding to Green functors, with no norms. In other words, a Green functor is the most naïve way to give a Mackey functor equivariant multiplicative structure.

Any incomplete Tambara functor $\underline{R}$ is a commutative ring-like object in the category of Mackey functors. It makes sense to talk about modules over $\underline{R}$, prime and maximal ideals of $\underline{R}$ [Nak12a], and to localize $\underline{R}$ at a multiplicatively closed system [Nak12b]. However, the commutative and homological algebra of incomplete Tambara functors has not been systematically pursued. My thesis develops aspects of the algebra of incomplete Tambara functors. A Hochschild–Kostant–Rosenberg (HKR) theorem for incomplete Tambara functors would enable new computations in rational equivariant algebraic K-theory (Section 3). A lemma in the proof of one version of the HKR theorem relies on the fact that the free $\mathbb{Z}$-algebra $\mathbb{Z}[x]$ is free as a $\mathbb{Z}$-module. This fact is surprisingly very rarely true for incomplete Tambara functors (Section 2).

2 Free incomplete Tambara functors are almost never flat

Here is a fact that we take for granted in commutative algebra. Consider the free $\mathbb{Z}$-algebra on a single generator $\mathbb{Z}[x]$. This algebra is also a free $\mathbb{Z}$-module on the countable basis $1, x, x^2, x^3, \ldots$. We often make use of this fact in homological algebra computations. In our preprint [HMQ21], Mike Hill, J.D. Quigley

[1] An example of this situation is Bousfield localization of genuine equivariant ring spectra, which does not necessarily preserve all of the norms [HH16].

and I investigate the analogous property for incomplete Tambara functors. Surprisingly, it doesn't always hold! This means that standard homological algebra techniques will need some modification to work with incomplete Tambara functors.

Question 2.1. When is a free incomplete Tambara functor free as a Mackey functor? When is it flat?

The Burnside Tambara functor $\underline{A}$ plays the role that the integers $\mathbb{Z}$ do for commutative algebras. Namely, $\underline{A}$ is the initial Tambara functor and the unit for the box product of Mackey functors, just as $\mathbb{Z}$ is the initial commutative ring and the unit for the tensor product of abelian groups. By forgetting some of the norms present in the Tambara functor $\underline{A}$, we can consider it as an incomplete Tambara functor for any indexing system $\mathcal{O}$. To keep track of which kinds of norms we have, we write this indexing system in the notation as a superscript: $\underline{A}^{\mathcal{O}}$.

For a given indexing system $\mathcal{O}$, there are many different free $\mathcal{O}$-Tambara functors on a single generator, depending on how the group acts on the generator. We write $\underline{A}^{\mathcal{O}}[x_{G/H}]$ for the free $\mathcal{O}$-Tambara functor on a single generator that lies in an orbit of shape G/H. This object is free in the sense that it satisfies the universal property that is the equivariant version of the universal property of $\mathbb{Z}[x]$: commutative ring homomorphisms from $\mathbb{Z}[x]$ to any commutative ring R are isomorphic to R.

Each free incomplete Tambara functor is parameterized both by an indexing system $\mathcal{O}$, specifying which norms are present, and an orbit G/H, specifying the action of G on the generator. We provide conditions on $\mathcal{O}$ and H such that the free incomplete Tambara functor is free as a Mackey functor.

Theorem 2.2 (Hill, Mehrle, Quigley [HMQ21])**.** *Let G be a solvable finite group with subgroup H and let $\mathcal{O}$ be an indexing system for G. The following are equivalent:*

(a) H is a normal subgroup of G, plus two easily verifiable combinatorial conditions on $\mathcal{O}$;

(b) the Mackey functor underlying the free $\mathcal{O}$-Tambara functor $\underline{A}^{\mathcal{O}}[x_{G/H}]$ is flat;

(c) the Mackey functor underlying the free $\mathcal{O}$-Tambara functor $\underline{A}^{\mathcal{O}}[x_{G/H}]$ is free.

The implications $(a) \implies (b)$ and $(a) \implies (c)$ do not require the solvable assumption.

If our free incomplete Tambara functor is not free as a Mackey functor, it might still be suitable for homological algebra if it is projective or flat instead. However, our theorem quashes that possibility in most cases: when G is solvable, the flat ones are exactly the free ones. One might wonder, then, how often the conditions in the theorem are satisfied.

Question 2.3. How often are free incomplete Tambara functors flat? More precisely, how often do the conditions in Theorem 2.2(a) hold?

The answer turns out to be less than we might hope. For example, consider the case of cyclic p-groups:

Group	total free incomplete Tambara functors for G	number which are free as Mackey functors	percent
$\{e\}$	1	1	100%
C_p	4	2	50%
C_{p^2}	15	4	$\approx$ 27%
C_{p^3}	56	9	$\approx$ 16%

As the depth of the subgroup lattice increases, the proportion of free incomplete Tambara functors which are free as Mackey functors only decreases. We observe this phenomenon for other families of groups as well, such as dihedral groups. The next theorem makes this rigorous, and gives us the slogan: "free incomplete Tambara functors are almost never flat."

Theorem 2.4 (Hill, Mehrle, Quigley [HMQ21])**.** *Pick any triple* $(G, H, \mathcal{O})$ *of a finite group* G *with subgroup* H *and indexing system* $\mathcal{O}$ *for* G*. Then the free incomplete Tambara functor* $\underline{A}^{\mathcal{O}}[x_{G/H}]$ *is free (or flat) with probability zero.*

At this point, the situation is looking pretty dire. The property that we want almost never holds! But there's still hope: by inverting a single element in the Burnside functor $\underline{A}$, we find that the situation changes dramatically.

Theorem 2.5 (Hill, Mehrle, Quigley [HMQ21])**.** *Let* $\mathcal{O}$ *be an indexing system. Let* $\underline{S}^{-1}\underline{A}^{\mathcal{O}}$ *be the incomplete Tambara functor obtained from* $\underline{A}^{\mathcal{O}}$ *by inverting the element* $[G] \in \underline{A}^{\mathcal{O}}(G/G)$*. Then for any subgroup* H *of* G*, the free incomplete Tambara functor* $\underline{S}^{-1}\underline{A}^{\mathcal{O}}[x_{G/H}]$ *is free as a Mackey functor.*

This theorem suggests that, as in classical algebra, working with derived functors will be easier after rationalization or localization. However, this is not without its caveats. This localization operation is quite brutal; it essentially destroys all information that is not already present in the data of the incomplete Tambara functor evaluated on G/e. One reason that this localization is so destructive is that inverting $[G]$ necessarily also inverts the classes $[G/H]$ for all subgroups H. So we are seeing two extremes here:

(1) Nothing is inverted, and almost nothing is free as a Mackey functor.

(2) $[G/H]$ is inverted for all H, and everything is free as a Mackey functor.

The in-between cases leave a lot of room to explore. For a fixed indexing system $\mathcal{O}$, what is the largest subgroup H such that all free $\mathcal{O}$-Tambara functors are free as Mackey functors after inverting $[G/H]$ and $[G/J]$ for all $J \geq H$? Can we predict a pattern given only the indexing system $\mathcal{O}$? Are there combinations of indexing system $\mathcal{O}$ and subgroup H such that inverting $[G/H]$ yields a nonzero proportion of free things less than "all of them"?

Goal 2.6. Understand how localizations of the Burnside Mackey functor $\underline{A}$ affect which free incomplete Tambara functors are free as Mackey functors.

3 A Hochschild–Kostant–Rosenberg theorem for incomplete Tambara functors

One of the most important tools we have for studying algebraic K-theory is the trace map [BoHM93, HM03, NS18]. For a ring A, the most basic form of the trace map is a map

$$K_*(A) \to HH_*(A),$$

where $HH_*(A)$ is the Hochschild homology of A.

If instead A is a ring with involution, a kind of C_2-action, we study its Real algebraic K-theory [HM15] or Hermitian K-theory [Sch10, CDH+21a, CDH+21b, CDH+21c]. In this situation, there is a trace map

$$KR_*(A) \to HHR_*(A)$$

whose target is Real hochschild homology [AGH21, DO19] – a kind of C_2-equivariant Hochschild homology.

Recently, there has been interest in equivariant algebraic K-theory of rings equipped with action by an arbitrary finite group [Mer17, Bar17]. For this equivariant algebraic K-theory, there is no trace map yet. Part of the problem is that there is no notion of equivariant Hochschild homology.

Question 3.1. Is there an equivariant Hochschild homology that will serve as the target of a trace map from equivariant K-theory?

One clue towards the answer comes from the Hochschild–Kostant–Rosenberg (HKR) theorem [HKR62]. The classical HKR theorem states that the Hochschild homology of a smooth k-algebra A is the same as the wedge powers of its Kähler differentials:

$$\mathrm{HH}_*(A/k) \cong \bigwedge^* \Omega^1_{A/k}. \tag{3.2}$$

In [Hil17, Lee19], Hill and Leeman define an $\underline{R}$-module $\Omega^{1,G}_{\underline{R}/\underline{S}}$ of *genuine Kähler differentials* for a morphism of incomplete Tambara functors $\underline{S} \to \underline{R}$. Taking $\underline{S}$ and $\underline{R}$ to be fixed point Tambara functors of a ring with G-action, this yields a notion of genuine Kähler differentials for rings with G-action as well. But the question remains: what should go on the left hand side of (3.2)?

Goal 3.3. Define Hochschild homology of incomplete Tambara functors so that an equivariant version of the HKR theorem (3.2) holds.

As a first pass, we might define Hochschild homology for incomplete Tambara functors by mimicking the classical construction. With this definition, J.D. Quigley and I have proved the following:

Theorem 3.4 (Mehrle, Quigley). *Let G be a finite group. Given a morphism of incomplete Tambara functors $\underline{S} \to \underline{R}$, there is a surjection of $\underline{R}$-modules*

$$\mathrm{HH}_1(\underline{R}/\underline{S}) \twoheadrightarrow \Omega^{1,G}_{\underline{R}/\underline{S}}. \tag{3.5}$$

When $\underline{S} \to \underline{R}$ is a morphism of Green functors, this is an isomorphism.

Nevertheless, when $\underline{S} \to \underline{R}$ is not a morphism of Green functors, (3.5) need not be an isomorphism.

Theorem 3.6 (Mehrle, Quigley). *For the free C_2-Tambara functor $\underline{R} = \underline{A}^{\mathcal{O}'}[x_{C_2/C_2}]$,*

$$\mathrm{HH}_1(\underline{R}/\underline{A}) \ncong \Omega^{1,G}_{\underline{R}/\underline{A}}.$$

There are several obvious directions to take Theorem 3.4 from here. First, we would like to extend this to a full HKR thoerem in degrees $n > 1$ for Green functors. Second, we would like to find the correct notion of Hochschild Homology such that (3.5) is an isomorphism for arbitrary incomplete Tambara functors instead of just Green functors. I will discuss each of these in turn.

Goal 3.7. Extend Theorem 3.4 to an HKR theorem for all degrees for Green functors.

A key lemma in the proof of the classical HKR theorem is the local-to-global principle: an A-module M is zero if and only if all of its localizations at prime ideals are. This lemma is useful in many other commutative algebra contexts. The study of ideals and localizations of Tambara functors was started by Nakaoka [Nak12a, Nak12b], and extended to all incomplete Tambara functors by Blumberg–Hill [BH18]. Incomplete Tambara functors have prime and maximal ideals, a prime ideal spectrum, and can be localized at a multiplicatively closed sub-functor. With J.D. Quigley and Jack Carlisle, I am working on the following:

Goal 3.8. Understand a local-to-global principle for Green functors and general incomplete Tambara functors.

Extending Theorem 3.4 to an arbitrary incomplete Tambara functor $\underline{R}$ runs into some obstacles. From calculations, it appears that $\mathrm{HH}_1(\underline{R})$ and $\Omega^{1,G}_{\underline{R}/\underline{A}}$ are not always the same unless $\underline{R}$ is a Green functor. Conceptually, this is because of the presence of norms in $\underline{R}$. Modules for an incomplete Tambara functor are modules over the underlying Green functor, so the Hochschild homology of $\underline{R}$ doesn't see the norms, while $\Omega^{1,G}$ does. To address this, we might change the notion of module that we work with. Strickland [Str12] defines a category of *genuine $\underline{R}$-modules* that contains the usual category of $\underline{R}$-modules (by contrast, these are called the *naïve modules*). When $\underline{R}$ is a Green functor, the genuine and naïve modules agree, but there are more genuine modules in general. Although the category of genuine modules is harder to work with, we hope that it will yield the correct version of equivariant Hochschild homology. To start, we must understand how to work with the category of genuine modules.

Goal 3.9. Understand homological algebra for genuine $\underline{R}$-modules over an incomplete Tambara functor $\underline{R}$.

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