Welcome! Let's calculate derivatives.

Math 1910: Calculus for Engineers

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Last Time	Today	Upcoming
Nothing	Introductions	HW Due 1 Sep.
	Derivatives Review	Quiz on 1 Sep.

Administrative

- All course information can be found on https://blackboard.cornell.edu/
- Section is review, practice problems, homework questions, etc.
- Occasionally there will be workshops with engineering applications
- Homework is due on Thursdays and graded for completion.
- Quizzes **every Thursday** with questions straight from homework.

Derivatives Review

- Given a function f, the **derivative** of f at the point a is defined by

$$f'(a) := \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- The line tangent to $(a, f(a))$ is $y - f(a) = f'(a)(x - a)$.

  - $(cf)' = cf'$ if c is a constant.
  - $(f+g)' = f' + g'$
  - **Product rule**: $(fg)' = f'g + fg'$.
  - **Quotient rule**: $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$.
  - **Chain rule**: $(f(g(x)))' = f'(g(x))g'(x)$.

*rhymes with "early"

- **Implicit differentiation** is used to compute $\frac{dy}{dx}$ when the variables x and y are related by an equation, such as $x^3 - y^3 = 4$.
- **The first derivative test:** If f is differentiable and c is a critical point, then the type of critical point can be found in the table.

Sign Change	Type of Critical Point
From $+$ to $-$	Local max
From $-$ to $+$	Local min

- A function f is **concave up** on (a, b) if f' is increasing, and **concave down** if f is decreasing. A **point of inflection** is a point $(c, f(c))$ where the concavity changes. We can use the first derivative test on f' to find the inflection points.

Problems

(1) Compute $\frac{dy}{dx}$.

(a) $15x^2 - 14x$

(b) $\frac{1-x^2}{(x^2+1)^2}$

(c) $\left(x^4 - 9x\right)^6$

(d) $\dfrac{\frac{1}{2\sqrt{x}} + 1}{2\sqrt{x + \sqrt{x}}}$

(e) $\sec^2(x)$

(f) $2\cos^2(x)(2\cos(2x) - 1)$

(g) $-\dfrac{\cot(x)\csc(x)\sec^2\left(\sqrt{\csc(x)+1}\right)}{2\sqrt{\csc(x)+1}}$

(h) Use implicit differentiation. $\dfrac{dy}{dx} = \dfrac{4 - 3x^2}{3y^2}$ when $y \neq 0$.

(i) Use implicit differentiation. $\dfrac{dy}{dx} = \dfrac{4x + y^2}{1 - 2xy}$

(j) Use implicit differentiation. $\dfrac{dy}{dx} = \dfrac{cos(x+y)}{1 - \cos(x+y)}$

(2) The points are $(-\frac{1}{3}, \frac{89}{27})$, $(3, 7)$.

(3) Find the critical points of f and determine if they are minima or maxima.

(a) maximum at $x = \frac{2}{3}$ and minimum at $x = 2$

(b) maximum at $x = \frac{-4}{5}$; minimum at $x = 0$

(c) maximum at $x = \frac{2}{5}$

(4) Find the points of inflection of the function f

(a) at $x = \frac{4}{3}$

(b) at $x = \frac{(2n+1)\pi}{2}$ for all integers n

(c) at $x = \pm\frac{2}{\sqrt{3}}$

(5) Whenever $a \geq 0$.