Review

§5.6 (Net Change as Integral of Rate of Change)

MATH 1910 Recitation
September 6, 2016

- The net change in a quantity $s(t)$ is equal to the $\boxed{\text{integral}}$ (1) of the rate of change of $s(t)$.
  - Net change between time $t = t_1$ and time $t = t_2$: $\boxed{s(t_2) - s(t_1)}$ (2)
  - Rate of change at time t: $\boxed{s'(t)}$ (3)
  - Integral of rate of change between time $t = t_1$ and time $t = t_2$: $\boxed{\int_{t_1}^{t_2} s'(t)\,dt}$ (4)
- For an object traveling in a straight line velocity $v(t)$, what is
  - it's displacement during the interval $[t_1, t_2]$? $\boxed{\int_{t_1}^{t_2} v(t)\,dt}$ (5)
  - the total distance it travelled during $[t_1, t_2]$? $\boxed{\int_{t_1}^{t_2} |v(t)|\,dt}$ (6)
- If $C(x)$ is the cost of producing x units of a commodity, then $C'(x)$ is the marginal cost and the cost of increasing production from a-many units to b-many units is $\boxed{\int_a^b C'(x)\,dx}$ (7)

(1) Find the displacement over the time interval $[1, 6]$ of a helicopter whose vertical velocity at time t is $v(t) = .02t^2 + t$ feet per second.

SOLUTION: Given $v(t) = \frac{1}{50}t^2 + t$ feet per second, the change in height over $[1, 6]$ is

$$\begin{aligned}\int_1^6 v(t)\,dt &= \int_1^6 \left(\frac{1}{50}t^2 + t\right)\,dt \\ &= \left(\frac{1}{150}t^3 + \frac{1}{2}t^2\right)\Big|_1^6 \\ &= \left(\frac{1}{150}6^3 + \frac{1}{2}6^2\right) - \left(\frac{1}{150}1^3 + \frac{1}{2}1^2\right) = \frac{284}{15} \approx 18.93\text{feet.}\end{aligned}$$

(2) A particle is moving along a straight line with velocity $v(t) = \cos t$ meters per second. Find

(a) the total displacement over the interval $[0, 4\pi]$, and

SOLUTION: Total displacement is given by $\int_0^{4\pi} \cos t\,dt = \sin t\Big|_0^{4\pi} = 0$ meters.

(b) the total distance travelled over the interval $[0, 4\pi]$.

SOLUTION: Total distance is given by

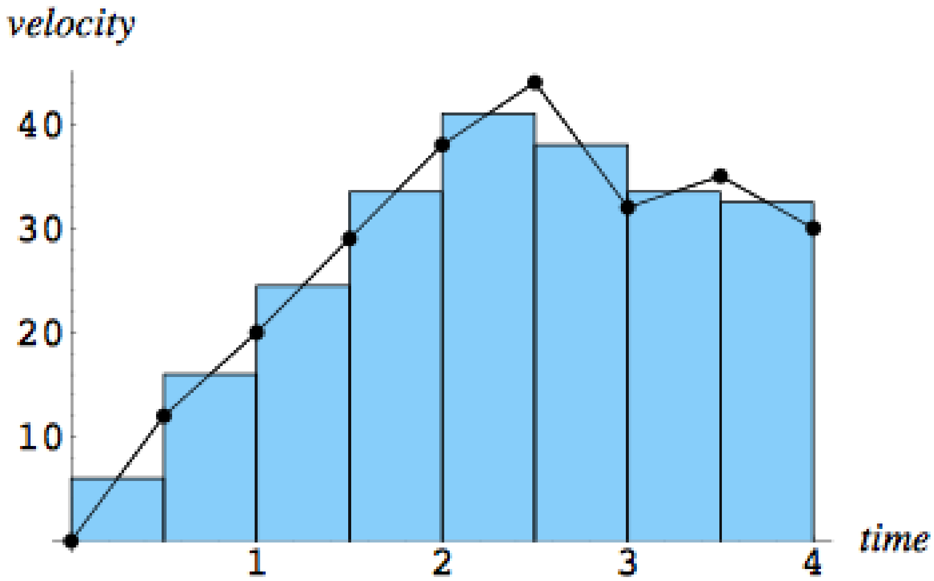
$$\begin{aligned}\int_0^{4\pi} |\cos t|\,dt &= \int_0^{\pi/2} \cos t\,dt + \int_{\pi/2}^{3\pi/2} -\cos t\,dt + \int_{3\pi/2}^{5\pi/2} \cos t\,dt + \int_{5\pi/2}^{7\pi/2} -\cos t\,dt + \int_{7\pi/2}^{4\pi} \cos t\,dt \\ &= \sin t\Big|_0^{\pi/2} - \sin t\Big|_{\pi/2}^{3\pi/2} + \sin t\Big|_{3\pi/2}^{5\pi/2} - \sin t\Big|_{5\pi/2}^{7\pi/2} + \sin t\Big|_{7\pi/2}^{4\pi} \\ &= 8\text{meters}\end{aligned}$$

(3) The velocity in feet per second of a car is recorded at half-second intervals in the table below.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$v(t)$	0	12	20	29	38	44	32	35	30

Use the average of the left-endpoint and right-endpoint approximations to estimate the total distance travelled over the time interval $[0, 4]$.

SOLUTION: First, draw a picture!



Let $\Delta x = 0.5$. Then

$$R_N = 0.5 \cdot (12 + 20 + 29 + 38 + 44 + 32 + 35 + 30) = 120\text{feet}$$

$$L_N = 0.5 \cdot (0 + 12 + 20 + 29 + 38 + 44 + 32 + 35) = 105\text{feet}$$

The average of R_N and L_N is 112.5 feet, which is a decent estimate for the distance travelled by the car over the interval.

(4) The heat capacity $C(T)$ of a substance is the amount of energy (in joules) required to raise the temperature of one gram of the substance by one degree Celsius when it's temperature is T. (The heat capacity depends on the substance's current temperature.)

(a) Determine the energy required to raise the temperature of one gram from T_1 to T_2

SOLUTION: Since $C(T)$ is the energy required to raise the temperature of one gram by one degree, when the temperature is T, the energy required to raise the temperature depends on the current temperature, which is always changing as energy is added. Hence, the total energy needed is the area under the graph of $C(T)$ between T_1 and T_2, which is $\displaystyle\int_{T_1}^{T_2} C(T)\,dT$.

(b) If a substance has heat capacity $C(T) = 6 + 0.2\sqrt{T}$, calculate the energy required to raise the temperature of one gram of the substance from $50°$ to $100°$ Celsius.

SOLUTION: If $C(T) = 6 + 0.2\sqrt{T} = 6 + \frac{1}{5}T^{1/2}$, then the energy required to raise the temperature from 50° C to 100° C is

$$\begin{aligned}
\int_{50}^{100} C(T)\,dT &= \int_{50}^{100} \left(6 + \frac{1}{5}t^{1/2}\right)\,dt \\
&= \left(6t + \frac{2}{15}t^{3/2}\right)\Big|_{50}^{100} \\
&= \left(6(100) + \frac{2}{15}(100)^{3/2}\right) - \left(6(50) + \frac{2}{15}(50)^{3/2}\right) \\
&= \frac{1300 - 100\sqrt{2}}{3} \approx 386.19\text{Joules}
\end{aligned}$$