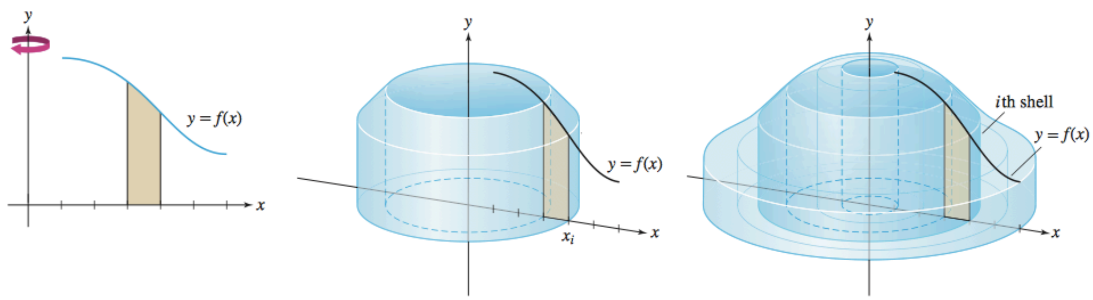
One-Page Review

§6.4 (Shell Method), §6.5 (Work and Energy)

MATH 1910 Recitation
September 27, 2016

(1) **Shell method:** When you rotate the region between two graphs around an axis, the segments **parallel** to the axis generate cylindrical shells. The volume V of the solid of revolution is the integral of the surface areas of shells.

$$V = \int \boxed{}^{(1)} .$$



(2) What is the volume of:

(a) The region between $f(x)$ and the x-axis ($f(x) \geq 0$) for $x \in [a, b]$ rotated around the y-axis?

$$V = \boxed{}^{(2)}$$

(b) The region between $f(x)$ and $g(x)$, $(f(x) \geq g(x) \geq 0)$ for $x \in [a, b]$ rotated around the y-axis?

$$V = \boxed{}^{(3)}$$

(c) The region between $f(x)$ and the x-axis $(f(x) \geq 0)$ for $x \in [a, b]$, rotated around the line $x = c$ $(c \leq a)$? What if $c \geq a$?

If $c \leq a, V = \boxed{}^{(4)}$ If $c \geq a, V = \boxed{}^{(5)}$

(3) The work W performed to move an object from a to b along the x-axis by applying a force of magnitude $F(x)$ is $W = \boxed{}^{(6)}$.

(4) To compute work against gravity, first decompose an object into N layers of equal thickness Δy, and then express the work performed on a thin layer as $L(y)\Delta y$, where

$$L(y) = g \times \text{density} \times \boxed{}^{(7)} \times \boxed{}^{(8)} .$$

Then the total work performed is $W = \boxed{}^{(9)}$.

PRACTICE PROBLEMS

§6.4 (Shell Method), §6.5 (Work and Energy)

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(1) Sketch the solid obtained by rotating the region underneath the graph of f over the interval about the given axis, and calculate its volume using the shell method.

(a) $f(x) = x^3$, $x \in [0, 1]$, about $x = 2$.

(b) $f(x) = x^3$, $x \in [0, 1]$ about $x = -2$.

(c) $f(x) = x^{-4}$, $x \in [-3, -1]$, about $x = 4$.

(d) $f(x) = \frac{1}{\sqrt{x^2+1}}$, $x \in [0, 2]$, about $x = 0$.

(2) Use the most convenient method (disk/washer or shell) to find the given volume of rotation.

(a) Region between $x = y(5 - y)$ and $x = 0$, rotated around the y-axis.

(b) Region between $x = y(5 - y)$ and $x = 0$, rotated about the x-axis.

(c) Region between $y = x^2$ and $x = y^2$, rotated about the y-axis.

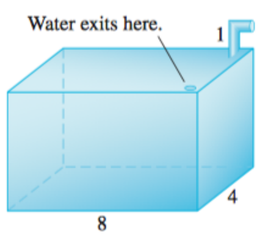
(d) Region between $y = x^2$ and $x = y^2$, rotated about $x = 3$.

(3) Calculate the work (in Joules) required to pump all of the water out of a full tank with the shape described. Distances are in meters, and the density of water is 1000 kg/m^3.

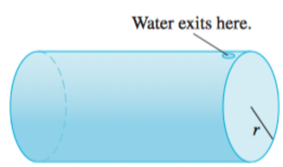
(a) A rectangular tank, with water exiting from a small hole at the top.

(b) A horizontal cylinder of length ℓ, where water exits from a small hole at the top.

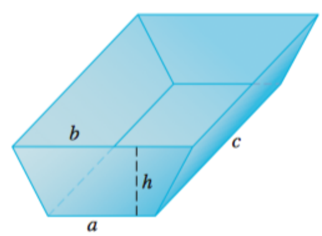
(c) A trough as in the picture, where water exits by pouring over the sides.



(a)



(b)



(c)

(4) Calculate the work required to lift a 6 meter chain with mass 18 kg over the side of a building.

(5) A 3 meter chain with mass density $\rho(x) = 2x(4 - x)$ kg/m lies on the ground. Calculate the work required to lift the chain from the front end so that its bottom is 2 meters above the ground.