One-Page Review

§7.7 (L'Hôspital's Rule), §7.8 (Inverse Trig)

MATH 1910 Recitation
October 13, 2016

(1) **L'Hôspital's Rule:** If $f(a) = g(a) = 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)} =$ [(1)] .

(2) What are all the indeterminate forms? There are seven of them. [(2)]

(3) To evaluate the limit involving an indeterminate form 0^0, 1^∞, or ∞^0, first [(3)] and then apply L'Hôspital's rule.

(4) Domain and range of inverse trigonometric functions.

(a) What is the domain of $\sin^{-1}$? [(4)] What is the range of $\sin^{-1}$? [(5)]

(b) What is the domain of $\cos^{-1}$? [(6)] What is the range of $\cos^{-1}$? [(7)]

(c) What is the domain of $\tan^{-1}$? [(8)] What is the range of $\tan^{-1}$? [(9)]

(d) What is the domain of $\cot^{-1}$? [(10)] What is the range of $\cot^{-1}$? [(11)]

(e) What is the domain of $\sec^{-1}$? [(12)] What is the range of $\sec^{-1}$? [(13)]

(f) What is the domain of $\csc^{-1}$? [(14)] What is the range of $\csc^{-1}$? [(15)]

(5) Derivatives of inverse trigonometric functions.

(a) $\frac{d}{dx} \sin^{-1}(x) =$ [(16)]

(b) $\frac{d}{dx} \tan^{-1}(x) =$ [(17)]

(c) $\frac{d}{dx} \sec^{-1}(x) =$ [(18)]

(d) $\frac{d}{dx} \cos^{-1}(x) =$ [(19)]

(e) $\frac{d}{dx} \cot^{-1}(x) =$ [(20)]

(f) $\frac{d}{dx} \csc^{-1}(x) =$ [(21)]

(6) Integrals of inverse trigonometric functions.

(a) $\int \frac{1}{\sqrt{1-x^2}}\, dx =$ [(22)]

(b) $\int \frac{1}{x^2+1}\, dx =$ [(23)]

(c) $\int \frac{1}{|x|\sqrt{x^2-1}} =$ [(24)]

PRACTICE PROBLEMS

§7.7 (L'Hôspital's Rule), §7.8 (Inverse Trig)

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(1) Use L'Hôspital's Rule to calculate the limit

(a) $\lim_{x\to\infty} \dfrac{x^{2/3}+3x}{x^{5/3}-x}$

(b) $\lim_{x\to\infty} \dfrac{3x^3+4x^2}{4x^3-7}$

(c) $\lim_{x\to 8} \dfrac{x^{5/3}-2x-16}{x^{1/3}-2}$

(d) $\lim_{x\to 0} \dfrac{\tan 4x}{\tan 5x}$

(e) $\lim_{x\to 0} \left(\cot x - \dfrac{1}{x}\right)$

(f) $\lim_{x\to \pi/2} \left(x-\dfrac{\pi}{2}\right)\tan x$

(g) $\lim_{x\to 0} \dfrac{x^2}{1-\cos x}$

(h) $\lim_{x\to 0} \left(\dfrac{1}{x^2} - \csc^2 x\right)$

(i) $\lim_{x\to 2} \dfrac{e^{x^2}-e^4}{x-2}$

(j) $\lim_{x\to 1} \dfrac{x(\ln x - 1)+1}{(x-1)\ln x}$

(k) $\lim_{x\to\infty} \dfrac{e^x - e}{\ln x}$

(l) $\lim_{x\to\infty} \dfrac{e^{2x}-1-x}{x^2}$

(m) $\lim_{x\to\infty} x^{1/x^2}$

(n) $\lim_{x\to 0^+} x^{\sin x}$

(2) Find the derivative.

(a) $y = \arctan(x/3)$

(b) $y = \sec^{-1}(x+1)$

(c) $y = e^{\cos^{-1}(x)}$

(d) $y = \csc^{-1}(x^{-1})$

(e) $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$

(f) $y = \frac{\cos^{-1}(x)}{\sin^{-1}(x)}$

(g) $y = \cos^{-1}(x + \sin^{-1}(x))$

(h) $y = \ln(\arcsin(x))$

(3) Evaluate the integral

(a) $\displaystyle\int_0^4 \frac{1}{4x^2+9}\,dx$

(b) $\displaystyle\int_{-1/5}^{1/5} \frac{1}{\sqrt{4-25x^2}}\,dx$

(c) $\displaystyle\int_{\sqrt{2}/4}^{1/2} \frac{1}{x\sqrt{16x^2-1}}\,dx$

(d) $\displaystyle\int \frac{1}{x\sqrt{x^4-1}}\,dx$

(e) $\displaystyle\int \frac{(x+1)}{\sqrt{1-x^2}}\,dx$

(f) $\displaystyle\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$

(g) $\displaystyle\int \frac{1}{\sqrt{5^{2x}1}}\,dx$