One-Page Review

MATH 1910 Recitation

§8.7 (Improper Integrals)
§8.8 (Probability and Integration)
§8.9 (Numerical Integration)

November 1, 2016

(1) The **improper integral** of f over $[a, \infty)$ is defined as

$$\int_a^\infty f(x)\,dx = \boxed{}\ (1)$$

We say that the improper integral **converges** if ______ (2), and **diverges** if ______ (3).

(2) The p**-integral** is $\int_a^\infty \frac{1}{x^p}\,dx$. If ______ (4), then this integral converges. If ______ (5), then it diverges.

(3) **Comparison test:** Assume $f(x) \geq g(x)$

If $\int_a^\infty f(x)\,dx$ converges, then $\int_a^\infty g(x)\,dx$ ______ (6).

If $\int_a^\infty g(x)\,dx$ diverges, then $\int_a^\infty f(x)\,dx$ ______ (7).

(4) If $p(x)$ is a **probability density function** or **PDF**, then $\int_{-\infty}^\infty p(x)\,dx =$ ______ (8)

(5) If X is a random variable with probability density function p, then the probability that X is between a and b is

$$P(a \leq X \leq b) = \boxed{}\ (9)$$

(6) The **mean** or **average value** of a random variable X with PDF $p(x)$ is ______ (10).

(7) There are three numerical approximations to $\int_a^b f(x)\,dx$:

(a) The **midpoint rule:** $M_N =$ ______ (11) $c_j = a + \left(j + \frac{1}{2}\right)\Delta x$.

(b) The **trapezoid rule:** $T_N =$ ______ (12)

(c) **Simpson's rule:** $S_N =$ ______ (13)

PRACTICE PROBLEMS

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(1) Determine whether the improper integral converges, and if it does, evaluate it.

(a) $\displaystyle\int_1^\infty \frac{1}{x^{20/19}}\,dx$

(b) $\displaystyle\int_{20}^\infty \frac{1}{t}\,dt$

(c) $\displaystyle\int_0^5 \frac{1}{x^{19/20}}\,dx$

(d) $\displaystyle\int_1^3 \frac{1}{\sqrt{3-x}}\,dx$

(e) $\displaystyle\int_{-2}^4 \frac{1}{(x+2)^{1/3}}\,dx$

(2) Find a constant C such that $p(x) = {}^{C}/_{(2+x)^3}$ is a probability density function on the interval $[2, 4]$.

(3) A company produces boxes of rice that are filled on average with 16 oz of rice. Due to machine error, the actual volume of rice is normally distributed with a standard deviation of 0.4 oz. Find $P(X > 17)$, the probability of a box having more than 17 oz of rice.

(4) Find the T_4 approximation for $\displaystyle\int_0^4 \sqrt{x}\,dx$.

(5) State whether M_{10} underestimates or overestimates $\displaystyle\int_1^4 \ln(x)\,dx$ and find a bound for the maximum possible error.