ONE-PAGE REVIEW

MATH 1910 Recitation

§11.1 (Sequences)

§11.2 (Summing an Infinite Series)

November 15, 2016

§11.3 (Convergence of Series with Positive Terms)

(1) A sequence **converges** to a limit L if for every number $\varepsilon > 0$, there is M such that ____ (1) for all $n > M$.

(2) If f is continuous and $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} f(a_n) =$ ____ (2).

(3) A sequences is called:

(a) ____ (3) if there exists M such that $|a_n| \leq M$ for all n.

(b) ____ (4) if either $a_n < a_{n+1}$ or $a_n > a_{n+1}$ for all n.

(4) If a sequence is both ____ (5) and ____ (6), then it converges.

(5) **The divergence test:** If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ ____ (7)

(6) A series that looks like $a_n = cr^n$ is called ____ (8) If $|r| \geq 1$, then it ____ (9). If $|r| < 1$, then

$$\sum_{n=K}^{\infty} cr^n = \text{____ (10)}$$

(7) **The integral test:** Assume that $a_n = f(n)$ for $n \geq M$. If $\int_M^{\infty} f(x)\,dx$ converges, then $\sum_{n=0}^{\infty} a_n$ ____ (11) If $\int_M^{\infty} f(x)\,dx$ diverges, then $\sum_{n=0}^{\infty} a_n$ ____ (12)

(8) **The comparison test:** If $a_n \leq b_n$, and $\sum_{n=0}^{\infty} b_n$ converges, then ____ (13) If $\sum_{n=0}^{\infty} b_n$ ____ (14), then ____ (15)

(9) **Limit comparison test:** Let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.

(a) If ____ (16), then $\sum a_n$ converges if and only if $\sum b_n$ converges.

(b) If ____ (17) and $\sum a_n$ converges, then $\sum b_n$ converges.

(c) If ____ (18) and $\sum b_n$ converges, then $\sum a_n$ converges.

Practice Problems

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(1) True or false?

(a) $\sum_{n=1}^{\infty} a_n = \sum_{k=1}^{\infty} a_k$

(b) $\sum_{n=4}^{6} a_n = \sum_{i=1}^{4} a_{i+3}$

(c) $\sum_{n=2}^{\infty} a_{n+3} = \sum_{n=5}^{\infty} a_n$

(d) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(e) If $\lim_{n\to\infty} a_n = \infty$, then $\sum_{n=1}^{\infty} a_n$ diverges.

(f) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty} a_n = \infty$.

(2) Determine the limit of the sequence or show that the sequence diverges.

(a) $a_n = \frac{e^n}{2^n}$ (b) $b_n = \frac{3n+1}{2n+4}$ (c) $c_n = \frac{\sqrt{n}}{\sqrt{n}+4}$

(3) Show that the sequence given by $a_n = \frac{3n^2}{n^2+2}$ is strictly increasing, and find an upper bound.

(4) Determine the limit of the series or show that the series diverges.

(a) $\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$

(b) $\sum_{n=0}^{\infty} e^n$

(c) $\sum_{n=1}^{\infty} \frac{1}{n}$.

(d) $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

(e) $\sum_{n=2}^{\infty} \frac{n^2}{n^4-1}$ (Limit Comparison Test)

(f) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2^n}$ (Comparison Test)

(g) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ (Integral Test)