One-Page Review

§11.6 (Power Series)
§11.7 (Taylor Series)

MATH 1910 Recitation
November 22, 2016

(1) An infinite series of the form $F(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$ is called a ______ (1) and c is called the ______ (2).

(2) The ______ (3) of $F(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$ is a constant R such that $F(x)$ converges absolutely for $|x-c| < R$ and diverges for $|x-c| > R$. If $F(x)$ converges for all x, then $R =$ ______ (4).

(3) To determine R, use ______ (5)

(4) $\sum_{n=0}^{\infty} x^n =$ ______ (6), with $R =$ ______ (7).

(5) The powerseries $T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$ is called the ______ (8) for $f(x)$. If $c = 0$, this is called a ______ (9).

(6) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!} =$ ______ (10)

(7) $(1+x)^a = 1 + \sum_{n=1}^{\infty} \binom{a}{n} x^n$ for $|x| < 1$, where $\binom{a}{n} =$ ______ (11)

Practice Problems

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(1) Show that all three of the following power series have the same radius of convergence, but different behavior at the endpoints.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{9^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{n9^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^2 9^n}$

(2) Use the geometric series formula to expand the function $\dfrac{1}{1+3x}$ in a power series with center $c = 0$ and determine radius of convergence.

(3) Write out the first four terms of the Taylor series $f(x)$ centered at $c = 3$ if $f(3) = 1$, $f'(3) = 2$, $f''(3) = 12$, $f'''(3) = 3$.

(4) Find the Taylor series of the following functions and determine the radius of convergence.

   (a) $f(x) = \sin(2x)$, centered at $x = 0$.

   (b) $f(x) = e^{4x}$, centered at $x = 0$.

   (c) $f(x) = x^2 e^{x^2}$, centered at $x = 0$.

   (d) $f(x) = \dfrac{1}{3x-2}$, centered at $c = -1$.

   (e) $f(x) = (1+x)^{1/3}$, centered at $c = 0$.

   (f) $f(x) = \sqrt{x}$, centered at $c = 4$.