PRELIM 2 REVIEW QUESTIONS

Math 1910 Section 205/209

(1) Calculate the following integrals.

(a) $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$

(b) $\displaystyle\int \sin^2(x)\cos^4(x)\,dx$

(c) $\displaystyle\int \sin^5(x)\cos^4(x)\,dx$

(d) $\displaystyle\int \tan^6(x)\sec^4(x)\,dx$

(e) $\displaystyle\int \cot^5(x)\csc^5(x)\,dx$

(f) $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\,dx$

(g) $\displaystyle\int \frac{\cosh(x)}{\sinh^2(x)}\,dx$

(h) $\displaystyle\int \sin^7(x)\cos^2(x)\,dx$

(i) $\displaystyle\int \frac{3x^2}{\sqrt{x^2-1}}\,dx$

(j) $\displaystyle\int \frac{\cosh(x)}{3\sinh(x)+4}\,dx$

(k) $\displaystyle\int \frac{x^2+11x}{(x-1)(x+1)^2}\,dx$

(l) $\displaystyle\int \frac{3x^2-2}{x-4}\,dx$

(m) $\displaystyle\int \coth^2(1-4t)\,dt$

(n) $\displaystyle\int \frac{1}{x^2+4x-5}\,dx$

(2) Find the volume of the solid obtained by rotating $y = x\sqrt{1-x^2}$ about the y-axis from $y=0$ to $y=1$.

(3) Find the arc length of the graph of $y = \tan(x)$ over the interval $[0, \pi/4]$.

(4) Suppose that a random variable X is distributed with density $p(x) = C\sqrt{1-x^2}$ on $[-1,1]$. Find C such that $p(x)$ defines a probability density function, and compute $P(-1/2 \le X \le 1)$.

(5) Find C such that $p(x) = Ce^{-x}e^{-e^{-x}}$ is a probability density function on $(-\infty,\infty)$.

(6) Suppose that a random variable X is distributed with density $p(x) = x^2e^{-x^2}$ on $(-\infty,\infty)$. Find the mean of X.

(7) Suppose that a random variable X is distributed with density $p(x) = \frac{1}{r}e^{-x/r}$ on $(0,\infty)$. Find the mean of X.

(8) Calculate T_6 for the integral $I = \displaystyle\int_0^2 x^3\,dx$.

(a) Is T_6 too large or too small? Explain graphically.

(b) Show that $K_2 = |f''(2)|$ may be used in the error bound and find a bound for the error.

(c) Evaluate I and check that the actual error is less than the bound computed in (b).

(9) Radium-226 has a half-life of 1590 years. Consider a mass of 100 mg of Radium-226.

(a) What is the mass of Radium remaining after 1000 years?

(b) When will the mass of Radium be 10 mg?

(10) Show that $\displaystyle\int_1^\infty e^{-x^2}\,dx$ converges using the Comparison Theorem.