

Prelim 1 is Tuesday, 3 October from 7:30pm to 9pm in 185 Statler Hall.
It covers chapter 5, chapter 6, and sections 7.1, 7.2, and 7.3.

Warning: These problems are by no means a comprehensive representation of the material that might appear on the exam. That is, there may be topics not covered by these problems that you are still responsible for knowing. Let these problems be a supplement to your preparation for the exam, but be sure to review other sources (e.g. your notes, homework assignments, and the textbook) as well.

1. Find the following definite or indefinite integrals.

(a) $\int \sec^2(2\theta) \tan(2\theta) d\theta$

(b) $\int \frac{x+2}{x+1} dx$

(c) $\int_{-2}^4 |(x-1)(x-3)| dx$

(d) $\int_1^3 [t] dt$, where $[x]$ is the greatest integer less than or equal to x

(e) $\int_0^{\pi/2} \sec^2(\cos \theta) \sin \theta d\theta$

(f) $\int \sin \theta \cos \theta e^{\cos^2 \theta + 1} d\theta$

2. Find the following derivatives.

(a) $G'(x)$, where $G(x) = \int_{-x}^{\sin x} t^3 dt$.

(b) $A'(\pi)$, where $A(x) = \int_2^x \frac{\cos t}{1+t} dt$.

3. Compute the area enclosed by the graphs of the given functions over the indicated interval.

(a) $y = x^3 - 2x^2 + x$, $y = x^2 - x$

(b) $y = \sin x$, $y = \cos x$, $0 \leq x \leq 5\pi/4$

4. Find the volume of the described solid.

(a) A solid whose horizontal cross sections at height y are circles of radius $(1+y)^{-2}$ for $0 \leq y \leq H$.

(b) The solid obtained by rotating the region bounded by $y = -x^2 + 4x - 3$, and $y = 0$ about the line $y = -1$.

(c) The solid obtained by rotating the region bounded by $y^2 = x^{-1}$, $x = 1$, and $x = 3$ about the line $y = -3$.

(d) The solid obtained by rotating the region bounded by $y = \sec x$, $y = \csc x$, $y = 0$, $x = 0$, and $x = \pi/2$ about the x -axis.

5. For $a > 0$, show that the volume obtained when the region between $y = a\sqrt{x - ax^2}$ and x -axis is rotated about the x -axis is independent of the constant a .

6. Find the average value of the following functions.

(a) $f(x) = (x + 1)(x^2 + 2x + 1)^{4/5}$ over the interval $[0, 4]$

(b) $f(x) = \sqrt{9 - x^2}$ over the interval $[0, 3]$.

7. Find the mass of a circular sheet of radius R if its radial density is given by $\rho(r) = \sin r^2$ (kg/m^2).

8. Find the work required to perform the following tasks.

(a) Stretch a spring from length 22 cm to 24 cm, given that the spring has length 15 cm at equilibrium and exerts a force of 50 N when stretched to 20 cm.

(b) Fill a spherical tank of radius 2 meters with water, where the water source is located 1 meter below the bottom of the tank. (The density of water is $1000 \text{ kg}/\text{m}^3$.)