PRELIM 2 REVIEW NAME: ______________________

Math 1910

Prelim 2 is Tuesday, 7 November from 7:30pm to 9pm. It covers sections 7.7, 7.8, 7.9, 8.1, 8.2, 8.3, 8.5, 8.7, 8.8, 9.1, 9.4.

Warning: *These problems are by no means a comprehensive representation of the material that might appear on the exam. That is, there may be topics not covered by these problems that you are still responsible for knowing. Let these problems be a supplement to your preparation for the exam, but be sure to review other sources (e.g. your notes, homework assignments, and the textbook) as well.*

(1) Find the derivatives of the following functions.

(a) $f(t) = (\sin^2 t)^t$

(b) $g(t) = \tanh^{-1}(e^x)$

(c) $h(t) = \sqrt{t^2 - 1}\,\sinh^{-1} t$

(2) Find the following definite or indefinite integrals.

(a) $\int (\sin x)(\cosh x)\, dx$

(b) $\int \frac{dt}{\cosh^2 t + \sinh^2 t}$

(c) $\int \frac{dx}{x + x^{-1}}$

(d) $\int \frac{dx}{x(x^2 - 1)^{3/2}}$

(e) $\int \frac{dx}{x^2 + 4x + 5}$

(f) $\int \frac{dx}{x^2 + 4x - 5}$

(g) $\int_0^{\pi/2} \cot\theta\, d\theta$

(h) $\int_1^{\infty} \frac{dx}{(x-2)(2x+3)}$

(i) $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$

(3) Calculate the following limits.

(a) $\lim_{n\to\infty} \left(1 + \frac{4}{n}\right)^n$

(b) $\lim_{t\to\infty} \frac{\ln(e^t + 1)}{t}$

(c) $\lim_{x\to 0} \frac{\tanh x - \sinh x}{\sin x - x}$

(4) (a) Find the arc length of $y = \cosh x$ over the interval $[a, b]$.

(b) Find the surface area obtained by rotating $y = \sin x$ about the x-axis for $0 \leq x \leq \pi$.

(c) Show that the arc length of $y = \ln(f(x))$ for $a \leq x \leq b$ is

$$\int_a^b \frac{\sqrt{f(x)^2 + f'(x)^2}}{f(x)}\,dx$$

(5) Let R be the region under the graph of $y = 1/(x+1)$ for $0 \leq x < \infty$. Which of the following quantities is finite? Calculate the ones that are.

(a) The area of R.

(b) The volume of the solid obtained by rotating R about the x-axis.

(c) The volume of the solid obtained by rotating R about the y-axis.

(6) Let

$$F(x) = x\sqrt{x^2 - 1} - 2\int_1^x \sqrt{t^2 - 1}\,dt.$$

Prove that $F(x)$ and $\cosh^{-1} x$ differ by a constant by showing that their derivatives are the same for all x. Then show that the constant must be zero by evaluating at $x = 1$, so that the functions $F(x)$ and $\cosh^{-1} x$ are in fact equal.