§5.1 APPROXIMATING AREAS
§5.2 DEFINITE INTEGRALS

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RAPID REVIEW

(1) Approximations to the area under the graph of f over the interval $[a, b]$:

Right-endpoint	Left-endpoint	Midpoint
$R_N = \Delta x \sum_{j=\boxed{\ }^{(1)}}^{\boxed{\ }^{(2)}} f(x_j)$	$L_N = \Delta x \sum_{j=\boxed{\ }^{(3)}}^{\boxed{\ }^{(4)}} f(x_j)$	$M_N = \Delta x \sum_{j=0}^{N-1} \boxed{\qquad}^{(5)}$

(2) If f is continuous on $[a, b]$, then the area A under the graph $y = f(x)$ is defined as

$$A := \boxed{\qquad\qquad}^{(6)}$$

(3) The **definite integral** is the $\boxed{\qquad}^{(7)}$ of the region between the graph of f and the x-axis. If f is $\boxed{\qquad}^{(8)}$ on $[a, b]$, then f is integrable over $[a, b]$.

(4) Some properties of definite integrals:

(a) $\int_a^b (f(x) + g(x))\,dx = \boxed{\qquad}^{(9)}$

(b) $\int_a^b Cf(x)dx = \boxed{\qquad}^{(10)}$

(c) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(d) $\int_a^b f(x)dx + \int_b^c f(x)dx = \boxed{\qquad}^{(11)}$

(5) Some formulas for computing integrals

(a) $\int_a^b C dx = \boxed{\qquad}^{(12)}$

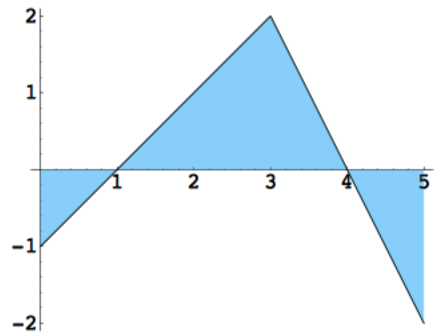
(b) $\int_0^b x dx = \boxed{\qquad}^{(13)}$

(c) $\int_0^b x^2 dx = \boxed{\qquad}^{(14)}$

(6) **Comparison Theorem:** If $f(x) \leq g(x)$ on $[a, b]$, then $\int_a^b f(x)\,dx \boxed{\quad}^{(15)} \int_a^b g(x)\,dx$.

PROBLEMS

(1) Use the graph of $g(x)$ given below to evaluate the following integrals.



(a) $\displaystyle\int_0^3 g(x)\,dx$

(b) $\displaystyle\int_3^5 g(x)\,dx$

(c) $\displaystyle\int_0^5 g(x)\,dx$

(2) Find a formula for R_N for $f(x) = 3x^2 - x + 4$ over the interval $[0, 1]$.

(3) Calculate $\displaystyle\int_2^5 (2x + 1)\,dx$ in three ways:

(a) As a limit $\lim_{N\to\infty} R_N$.

(b) Using geometry, interpreting this as the area under a graph.

(c) Using the properties of the integral.

(4) Use the basic properties of the integral to calculate the following.

(a) $\displaystyle\int_1^4 6x^2\,dx$

(b) $\displaystyle\int_{-2}^3 (3x + 4)\,dx$

(c) $\displaystyle\int_1^3 |2x - 4|\,dx$

(5) Evaluate $\displaystyle\lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \sqrt{1 - \left(\frac{j}{N}\right)^2}$ by interpreting the limit as an area.