§6.4: SHELL METHOD
§6.5: WORK AND ENERGY

NAME: ______________________

Math 1910

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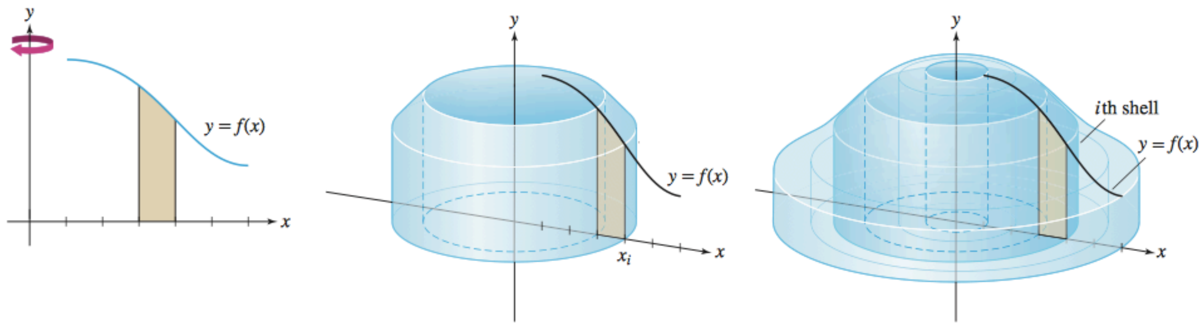
ONE-PAGE REVIEW

(1) **Shell Method**: When you rotate the region between two graphs around an axis, the segments **parallel** to the axis generate cylindrical shells. The volume V of the solid of revolution is the integral of the surface areas of the shells.

$$V = \int \boxed{}^{(1)} .$$



(2) What is the volume of

(a) the region between $f(x)$, $f(x) \geq 0$, and the x-axis for $x \in [a, b]$, rotated around the y-axis?

$$V = \boxed{}^{(2)}$$

(b) the region between $f(x)$ and $g(x)$, $f(x) \geq g(x) \geq 0$, for $x \in [a, b]$ rotated around the y-axis?

$$V = \boxed{}^{(3)}$$

(c) the region between $f(x)$, $f(x) \geq 0$ and the x-axis for $x \in [a, b]$, rotated around the line c?

If $c \leq a$, $V = \boxed{}^{(4)}$ If $c \geq a$, $V = \boxed{}^{(5)}$

(3) The work W performed to move an object from a to b along the x-axis by applying a force of magnitude $F(x)$ is $W = \int_a^b F(x)\, dx$.

(4) To compute work against gravity, first decompose an object into N layers of equal thickness Δy, and then express the work performed on a thin layer as $L(y)\Delta y$, where

$$L(y) = g \times \text{density} \times \text{area of } y \times \text{distance lifted}.$$

The total work performed to lift the object from height a to height b is $W = \boxed{}^{(6)}$.

PROBLEMS

(1) Sketch the solid obtained by rotating the region underneath the graph of f over the interval about the given axis, and calculate its volume using the shell method.

(a) $f(x) = x^3$, $x \in [0, 1]$, about $x = 2$.

(b) $f(x) = x^3$, $x \in [0, 1]$, about $x = -2$.

(c) $f(x) = \frac{1}{\sqrt{x^2+1}}$, $x \in [0, 2]$, about $x = 0$.

(2) Use the most convenient method (disk/washer or shell) to find the given volume of rotation.

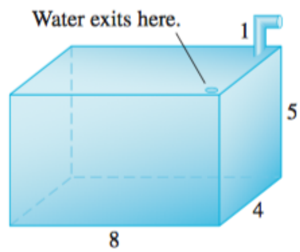
(a) Region between $x = y(5 - y)$ and $x = 0$, rotated about the y-axis.

(b) Region between $x = y(5 - y)$ and $x = 0$, rotated around the x-axis.

(c) Region between $y = x^2$ and $x = y^2$, rotated about $x = 3$.

(3) Calculate the work (in Joules) required to pump all of the water out of a full tank with the shape described. Distances are in meters, and the density of water is 1000 $\mathrm{kg/m^3}$.

(a) A rectangular tank, with water exiting from a small hole in the top.



(b) A trough as in the picture, where the water exits by pouring over the sides.