

§7.1: EXPONENTIAL FUNCTIONS

§7.2: INVERSE FUNCTIONS

§7.3 LOGARITHMS

Math 1910

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ONE-PAGE REVIEW

(1) $f(x) = b^x$ is increasing if ⁽¹⁾ and decreasing if ⁽²⁾.

(2) The derivative of $f(x) = b^x$ is $\frac{d}{dx} b^x =$ ⁽³⁾.

(3) $\frac{x^x}{e} =$ ⁽⁴⁾ and $\frac{x^{f(x)}}{e} =$ ⁽⁵⁾ and $\frac{x^{kx+b}}{e} =$ ⁽⁶⁾.

(4) $\int e^x dx =$ ⁽⁷⁾ and $\int e^{kx+b} =$ ⁽⁸⁾ for constants k, b .

(5) A function f with domain D is **one to one** if ⁽⁹⁾

(6) Let f have domain D and range R . The **inverse** f^{-1} is the unique function with domain R and range D such that ⁽¹⁰⁾.

(7) The inverse of f exists if and only if f is ⁽¹¹⁾ on its domain.

(8) **Horizontal Line Test:** f is one-to-one if and only if every horizontal line ⁽¹²⁾

(9) To find the inverse function, solve $y = f(x)$ for ⁽¹³⁾ in terms of ⁽¹⁴⁾.

(10) The graph of f^{-1} is obtained by ⁽¹⁵⁾ the graph of f through the line ⁽¹⁶⁾.

(11) If f is differentiable and one-to-one with inverse g , then for x such that $f'(g(x)) \neq 0$,

$$g'(x) = \frac{1}{f'(g(x))}.$$

(12) The inverse of $f(x) = b^x$ is ⁽¹⁷⁾.

(13) Logarithm Rules

(a) $\log_b(1) =$ ⁽¹⁸⁾ and $\log_b(b) =$ ⁽¹⁹⁾.

(b) $\log_b(xy) =$ ⁽²⁰⁾ and $\log_b\left(\frac{x}{y}\right) =$ ⁽²¹⁾

(c) **Change of Base:** $\frac{\log_a(x)}{\log_a(b)} =$ ⁽²²⁾.

(d) $\log_b(x^n) =$ ⁽²³⁾.

(14) $\frac{x}{\ln}(x) =$ ⁽²⁴⁾ and $\frac{x}{\log_b}(x) =$ ⁽²⁵⁾

(15) $\int \frac{1}{x} dx =$ ⁽²⁶⁾.

PROBLEMS

(1) Calculate the derivative.

(a) $f(x) = 7e^{2x} + 3e^{4x}$

(b) $f(x) = e^{e^x}$

(c) $f(x) = 3^x$

(d) $f(t) = \frac{1}{1 - e^{-3t}}$

(e) $f(t) = \cos(te^{-2t})$

(f) $\int_4^{e^x} \sin t \, dt$

(g) $f(x) = x \ln x$

(h) $f(x) = \ln(x^5)$

(i) $f(x) = \ln(\sin(x) + 1)$

(j) $f(x) = e^{\ln(x)^2}$

(k) $f(x) = \log_a(\log_b(x))$

(l) $f(x) = 16^{\sin x}$

(2) Calculate the integral.

(a) $\int \frac{7}{x} \, dx$

(b) $\int e^{4x} \, dx$

(c) $\int \frac{\ln x}{x} \, dx$

(d) $\int \frac{1}{9x-3} \, dx$

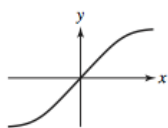
(e) $\int_2^3 (e^{4t-3}) \, dt$

(f) $\int e^t \sqrt{e^t + 1} \, dt$

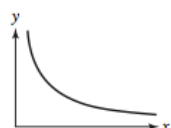
(g) $\int e^x \cos e^x \, dx$

(h) $\int \tan(4x + 1) \, dx$

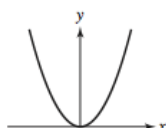
(3) For each function shown below, sketch the graph of the inverse.



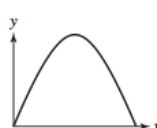
(A)



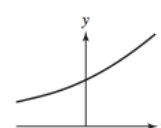
(B)



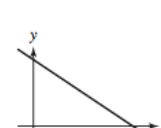
(C)



(D)



(E)



(F)

(4) Calculate $g(b)$ and $g'(b)$, where g is the inverse of f .

(a) $f(x) = x + \cos x$, $b = 1$.

(b) $f(x) = 4x^3 - 2x$, $b = -2$.

(c) $f(x) = \sqrt{x^2 + 6x}$ for $x \geq 0$, $b = 4$.

(d) $f(x) = \frac{1}{x+1}$, $b = \frac{1}{4}$.

(5) Which of the following statements are true and which are false? If false, modify the statement to make it correct.

(a) If f is increasing, then f^{-1} is increasing.

(b) If f is concave up, then f^{-1} is concave up.

(c) If f is odd then f^{-1} is odd.

(d) Linear functions $f(x) = ax + b$ are always one-to-one.

(e) $f(x) = \sin(x)$ is one-to-one.