§7.7: L'Hôpital's Rule
§7.8 Inverse Trig

Name: ______________________

Math 1910

October 12, 2017

One-page Review

(1) **L'Hôspital's Rule:** If $f(a) = g(a) = 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \boxed{\qquad\qquad}$ (1) .

(2) What are all the indeterminate forms? There are seven of them. $\boxed{\qquad\qquad\qquad\qquad}$ (2)

(3) To evaluate the limit involving an indeterminate form 0^0, 1^∞, or ∞^0, first take the logarithm and then apply L'Hôspital's rule.

(4) Domain and range of inverse trigonometric functions.

Function	Domain	Range
$\sin^{-1}(x)$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}(x)$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\tan^{-1}(x)$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$
$\cot^{-1}(x)$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1}(x)$	$(-\infty, -1) \cup (1, \infty)$	$[0, \pi/2) \cup (\pi/2, \pi]$
$\csc^{-1}$	$(-\infty, -1) \cup (1, \infty)$	$[-\pi/2, 0) \cup (0, \pi/2]$

(5) Derivatives and integrals involving inverse trigonometric functions.

$f(x)$	$\frac{d}{dx} f(x)$
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}(x)$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1}(x)$	$\frac{1}{x^2+1}$
$\cot^{-1}(x)$	$\frac{-1}{x^2+1}$
$\sec^{-1}(x)$	$\frac{1}{\lvert x\rvert\sqrt{x^2+1}}$
$\csc^{-1}(x)$	$\frac{-1}{\lvert x\rvert\sqrt{x^2+1}}$

$f(x)$	$\int f(x)\,dx$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x) + C$
$\frac{1}{x^2+1}$	$\tan^{-1}(x) + C$
$\frac{1}{\lvert x\rvert\sqrt{x^2+1}}$	$\sec^{-1}(x) + C$

PROBLEMS

(1) Use L'Hôspital's Rule to calculate the limit

(a) $\lim_{x\to\infty} \dfrac{3x^3+4x^2}{4x^3-7}$

(b) $\lim_{x\to 8} \dfrac{x^{5/3}-2x-16}{x^{1/3}-2}$

(c) $\lim_{x\to 0} \left(\dfrac{1}{x^2}-\csc^2 x\right)$

(d) $\lim_{x\to\infty} \dfrac{e^x-e}{\ln x}$

(e) $\lim_{x\to\infty} x^{1/x^2}$

(f) $\lim_{x\to 0^+} x^{\sin x}$

(2) Find the derivative.

(a) $y = \arctan(x/3)$

(b) $y = \sec^{-1}(x+1)$

(c) $y = e^{\cos^{-1}(x)}$

(d) $y = \csc^{-1}(x^{-1})$

(e) $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$

(f) $y = \frac{\cos^{-1}(x)}{\sin^{-1}(x)}$

(g) $y = \cos^{-1}(x+\sin^{-1}(x))$

(h) $y = \ln(\arcsin(x))$

(3) Evaluate the integral

(a) $\displaystyle\int_0^4 \frac{1}{4x^2+9}\,dx$

(b) $\displaystyle\int_{-1/5}^{1/5} \frac{1}{\sqrt{4-25x^2}}\,dx$

(c) $\displaystyle\int_{\sqrt{2}/4}^{1/2} \frac{1}{x\sqrt{16x^2-1}}\,dx$

(d) $\displaystyle\int \frac{1}{x\sqrt{x^4-1}}\,dx$

(e) $\displaystyle\int \frac{(x+1)}{\sqrt{1-x^2}}\,dx$

(f) $\displaystyle\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$

(g) $\displaystyle\int \frac{1}{\sqrt{5^{2x}-1}}\,dx$