§7.8: Inverse Trig
§7.9: Hyperbolic Trigonometry
§8.1: Integration by Parts

NAME: _____

Math 1910 October 17, 2017

One-page Review

(1) $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$ $\quad \cosh(x) = \dfrac{e^x + e^{-x}}{2}$ $\quad \tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$

$\coth(x) = \dfrac{\cosh(x)}{\sinh(x)}$ $\quad \operatorname{sech}(x) = \dfrac{1}{\cosh(x)}$ $\quad \operatorname{csch}(x) = \dfrac{1}{\sinh(x)}$

(2) Derivatives of hyperbolic trigonometric functions

$$\begin{aligned} \frac{d}{dx}\sinh(x) &= \cosh(x) & \frac{d}{dx}\cosh(x) &= \sinh(x) \\ \frac{d}{dx}\tanh(x) &= \operatorname{sech}^2(x) & \frac{d}{dx}\coth(x) &= -\operatorname{csch}^2(x) \\ \frac{d}{dx}\operatorname{sech}(x) &= -\operatorname{sech}(x)\tanh(x) & \frac{d}{dx}\operatorname{csch}(x) &= -\operatorname{csch}(x)\coth(x) \end{aligned}$$

(3) Integrals of hyperbolic trigonometric functions

$$\begin{aligned} \int \sinh(x)\,dx &= \cosh(x) + C & \int \cosh(x)\,dx &= \sinh(x) + C \\ \int \operatorname{sech}^2(x)\,dx &= \tanh(x) + C & \int \operatorname{csch}^2(x)\,dx &= -\coth(x) + C \\ \int \operatorname{sech}(x)\tanh(x)\,dx &= -\operatorname{sech}(x) + C & \int \operatorname{csch}(x)\coth(x)\,dx &= -\operatorname{csch}(x) + C \end{aligned}$$

(4) **Integration by parts**

$$\int u\,dv = \boxed{}^{(1)}$$

(5) (Repeat from last Thursday) Derivatives and integrals involving inverse trigonometric functions.

$$\begin{aligned} \tfrac{d}{dx}\sin^{-1}(x) &= \frac{1}{\sqrt{1-x^2}} \\ \tfrac{d}{dx}\cos^{-1}(x) &= \frac{-1}{\sqrt{1-x^2}} \\ \tfrac{d}{dx}\tan^{-1}(x) &= \frac{1}{x^2+1} \\ \tfrac{d}{dx}\cot^{-1}(x) &= \frac{-1}{x^2+1} \\ \tfrac{d}{dx}\sec^{-1}(x) &= \frac{1}{|x|\sqrt{x^2+1}} \\ \tfrac{d}{dx}\csc^{-1}(x) &= \frac{-1}{|x|\sqrt{x^2+1}} \end{aligned}$$

$$\begin{aligned} \int \frac{1}{\sqrt{1-x^2}}\,dx &= \sin^{-1}(x) + C \\ \int \frac{1}{x^2+1}\,dx &= \tan^{-1}(x) + C \\ \int \frac{1}{|x|\sqrt{x^2+1}}\,dx &= \sec^{-1}(x) + C \end{aligned}$$

PROBLEMS

(1) Simplify $\sinh(\ln x)$ and $\tanh(\frac{1}{2}\ln(x))$.

(2) Find the derivative.

(a) $y = \ln(\cosh(x))$.

(b) $y = \operatorname{sech}(x)\coth(x)$.

(3) Evaluate the integral.

(a) $\int \cosh(2x)\,dx$

(b) $\int \tanh(3t)\operatorname{sech}(3t)\,dt$

(c) $\int \frac{\cosh(x)}{3\sinh(x)+4}\,dx$

(d) $\int xe^{-x}\,dx$

(e) $\int x^3 e^{x^2}\,dx$.

(f) $\int_1^3 \ln x\,dx$.

(4) Find the volume of the solid obtained by revolving $y = \cos x$ for $0 \le x \le \pi/2$ around the y-axis.

(5) (Repeat from last Thursday) Evaluate the integral.

(a) $\int_0^4 \frac{1}{4x^2+9}\,dx$

(b) $\int_{-1/5}^{1/5} \frac{1}{\sqrt{4-25x^2}}\,dx$

(c) $\int_{\sqrt{2}/4}^{1/2} \frac{1}{x\sqrt{16x^2-1}}\,dx$

(d) $\int \frac{1}{x\sqrt{x^4-1}}\,dx$

(e) $\int \frac{(x+1)}{\sqrt{1-x^2}}\,dx$

(f) $\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$

(g) $\int \frac{1}{\sqrt{5^{2x}-1}}\,dx$