§8.2: Trigonometric Integrals
§8.3: Trigonometric Substitution
§8.5: Partial Fractions

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Rapid Review

(1) **Power-reducing identitites**

$$\cos^2(x) = \frac{1+\cos(2x)}{2}, \qquad \sin^2(x) = \frac{1-\sin(2x)}{2}$$

(2) **Completing the square.** If you have an integral with a $1/\sqrt{ax^2+bx+c}$ in it, you need to complete the square. Rewrite

$$ax^2+bx+c = a(x-h)^2+k$$

where

$$h = \boxed{}^{(1)}, \qquad k = \boxed{}^{(2)}$$

(3) **Partial Fractions:** if you have an expression that looks like

$$\frac{f(x)}{(x-a_1)(x-a_2)\cdots(x-a_n)}$$

where there are no repeats in the a_i's, then you can write

$$\frac{f(x)}{(x-a_1)(x-a_2)\cdots(x-a_n)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \ldots + \frac{A_n}{x-a_n}$$

If there are repeats in the a_i's, then $(x-a)^n$ contributes

$$\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_n}{(x-a)^n}.$$

And $(x^2+b)^n$ contributes

$$\frac{A_1x+B_1}{x^2+b} + \frac{A_2x+B_2}{(x^2+b)^2} + \ldots + \frac{A_nx+B_n}{(x^2+b)^n}.$$

PROBLEMS

(1) For each of the following integrals, should you use substitution, integration by parts, trig substitution, partial fractions, or something else?

(a) $\int \ln(x)\,dx$

(b) $\int \sqrt{4x^2-1}\,dx$

(c) $\int \frac{x}{\sqrt{12-6x-x^2}}\,dx$

(d) $\int \sin^3(x)\cos^3(x)\,dx$

(e) $\int x\sec^2(x)\,dx$

(f) $\int \frac{1}{\sqrt{9-x^2}}\,dx.$

(g) $\int x^2\sqrt{x+1}\,dx$

(h) $\int \frac{1}{(x+1)(x+2)^3}\,dx$

(i) $\int \frac{1}{(x+12)^4}\,dx$

(2) Evaluate the integral.

(a) $\int \frac{1}{\sqrt{x^2+9}}\,dx$

(b) $\int x\sqrt{x^2-5}\,dx.$

(c) $\int \frac{3x+5}{x^2-4x-5}\,dx$

(d) $\int e^{2x}\cos(x)\,dx$

(e) $\int \cos^2\theta\sin^2\theta\,d\theta$

(f) $\int \cos(x)\sin^5(x)\,dx$

(g) $\int \frac{1}{x(x-1)^2}\,dx$

(h) $\int \cos^2(4x)\,dx$

(i) $\int \frac{3}{(x+1)(x^2+x)}\,dx$

(j) $\int (\ln x+1)\sqrt{(x\ln x)^2+1}\,dx$