§8.7: Improper Integrals
§8.8: Probability and Integration

Name: ______________________

Math 1910

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One-Page Review

(1) The **improper integral** of f over $[a, \infty)$ is defined as

$$\int_a^\infty f(x)\,dx = \boxed{}^{(1)}$$

We say that the improper integral **converges** if $\boxed{}^{(2)}$, and **diverges** if $\boxed{}^{(3)}$.

(2) The p-**integral** is $\displaystyle\int_a^\infty \frac{1}{x^p}\,dx$. If $\boxed{}^{(4)}$, then this integral converges. If $\boxed{}^{(5)}$, then it diverges.

(3) **Comparison test:** Assume $f(x) \geq g(x)$

- If $\displaystyle\int_a^\infty f(x)\,dx$ converges, then $\displaystyle\int_a^\infty g(x)\,dx$ $\boxed{}^{(6)}$.
- If $\displaystyle\int_a^\infty g(x)\,dx$ diverges, then $\displaystyle\int_a^\infty f(x)\,dx$ $\boxed{}^{(7)}$.

(4) If $p(x)$ is a **probability density function** or **PDF**, then $\displaystyle\int_{-\infty}^\infty p(x)\,dx = \boxed{}^{(8)}$

(5) If X is a random variable with probability density function p, then the probability that X is between a and b is

$$P(a \leq X \leq b) = \boxed{}^{(9)}$$

(6) The **mean** or **average value** of a random variable X with PDF $p(x)$ is $\boxed{}^{(10)}$.

(7) The **normal distribution** with mean μ and standard deviation σ is the distribution with density

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

PROBLEMS

(1) Determine whether the improper integral converges, and if it does, evaluate it.

- $\displaystyle\int_1^\infty \frac{1}{x^{20/19}}\,dx$
- $\displaystyle\int_{20}^\infty \frac{1}{t}\,dt$
- $\displaystyle\int_0^5 \frac{1}{x^{19/20}}\,dx$
- $\displaystyle\int_1^3 \frac{1}{\sqrt{3-x}}\,dx$
- $\displaystyle\int_{-2}^4 \frac{1}{(x+2)^{1/3}}\,dx$

(2) Find a constant C such that $p(x) = {}^{C}/_{(2+x)^3}$ is a probability density function on the interval $[2, 4]$.

(3) A company produces boxes of rice that are filled on average with 16 oz of halloween candy. Due to the witch's curse placed on the founder of the company many generations ago, the actual volume of candy is normally distributed with a standard deviation of 0.4 oz. Find $P(X > 17)$, the probability of a box having more than 17 oz of candy.