§8.9: NUMERICAL INTEGRATION
§9.1: ARC LENGTH AND SURFACE AREA
§9.4: TAYLOR POLYNOMIALS NAME: ____________________

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ONE-PAGE REVIEW

(1) There are three numerical approximations to $\int_a^b f(x)\,dx$:

(a) The **midpoint rule:** $M_N = \Delta x\,(f(c_1) + \ldots + f(c_N))\,,\, c_j = a + \left(j + \frac{1}{2}\right)\Delta x.$

(b) The **trapezoid rule:** $T_N = \frac{1}{2}\Delta x\,(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{N-1} + y_N)$

(c) **Simpson's rule:** $S_N = \frac{1}{3}\Delta x\,(y_0 + 4y_1 + 2y_2 + \ldots + 4y_{N-3} + 2y_{N-2} + 4y_{N-1} + y_N)$

(2) The **arc length** of $f(x)$ on the interval $[a, b]$ is [ ] (1)

(3) The **surface area** of the surface obtained by rotating the graph of $f(x)$ around the x-axis for $a \leq x \leq b$ is [ ] (2)

(4) The n**-th Taylor Polynomial** centered at $x = a$ for the function f is

$T_n(x) =$ [ ] (3)

(5) The **error for the** n**-th Taylor Polynomial** is

$|T_n(x) - f(x)| \leq$ [ ] (4) ,

where K is the maximum of $|f^{(n+1)}(u)|$ over all u between a and x.

(6) **Taylor's Theorem** says that

$R_n(x) = T_n(x) - f(x) =$ [ ] (5)

PROBLEMS

(1) Find the T_4 approximation for $\int_0^4 \sqrt{x}\, dx$.

(2) State whether M_{10} underestimates or overestimates $\int_1^4 \ln(x)\, dx$.

(3) For the curve curve $y = \ln(\cos x)$ over the interval $[0, \pi/4]$, set up an integral to calculate:

   (a) the arc length.

   (b) the surface area when rotated around the x-axis.

(4) Approximate the arc length of the curve $y = \sin(x)$ over the interval $[0, \pi/2]$ using the midpoint approximation M_8.

(5) Find the Taylor polynomials $T_2(x)$ and $T_3(x)$ for $f(x) = \frac{1}{1+x}$ centered at $a = 1$.

(6) Find n such that $|T_n(1.3) - \sqrt{1.3}| \leq 10^{-6}$, where T_n is the Taylor polynomial for $\sqrt{x}$ at $a = 1$.