§11.1: SEQUENCES
§11.2 SUMMING AN INFINITE SERIES
§11.3 SERIES WITH POSITIVE TERMS

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ONE-PAGE REVIEW

(1) If f is continuous and $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} f(a_n) = f(L)$.

(2) A sequences is called:

- (a) **bounded** if there exists M such that $|a_n| \leq M$ for all n.
- (b) **monotone** if either $a_n < a_{n+1}$ or $a_n > a_{n+1}$ for all n.

If a sequence is both bounded and monotone, then it converges.

(3) **The divergence test:** If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

(4) A series that looks like $a_n = cr^n$ is called **geometric.**

- (a) If $|r| \geq 1$, then it diverges.
- (b) If $|r| < 1$, then $\sum_{n=K}^{\infty} cr^n = \frac{cr^K}{1-r}$

(5) **The integral test:** Assume that $a_n = f(n)$ for $n \geq M$.

- (a) If $\int_M^{\infty} f(x)\,dx$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.
- (b) If $\int_M^{\infty} f(x)\,dx$ diverges, then $\sum_{n=0}^{\infty} a_n$ diverges.

(6) **The comparison test:**

- (a) If $a_n \leq b_n$, and $\sum_{n=0}^{\infty} b_n$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.
- (b) If $\sum_{n=0}^{\infty} b_n$ diverges, then $\sum_{n=0}^{\infty} b_n$ diverges.

(7) **Limit comparison test:** If $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists and is not zero, then $\sum_{n=0}^{\infty} b_n$ converges if and only if $\sum_{n=0}^{\infty} a_n$ converges.

PROBLEMS

(1) True or false?

(a) $\sum_{n=1}^{\infty} a_n = \sum_{k=1}^{\infty} a_k$

(b) $\sum_{n=4}^{6} a_n = \sum_{i=1}^{4} a_{i+3}$

(c) $\sum_{n=2}^{\infty} a_{n+3} = \sum_{n=5}^{\infty} a_n$

(d) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(e) If $\lim_{n\to\infty} a_n = \infty$, then $\sum_{n=1}^{\infty} a_n$ diverges.

(f) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n\to\infty} a_n = \infty$.

(2) Determine the limit of the sequence or show that the sequence diverges.

(a) $a_n = \frac{e^n}{2^n}$

(b) $b_n = \frac{3n+1}{2n+4}$

(c) $c_n = \frac{\sqrt{n}}{\sqrt{n}+4}$

(3) Show that the sequence given by $a_n = \frac{3n^2}{n^2+2}$ is strictly increasing, and find an upper bound.

(4) Determine the limit of the series or show that the series diverges.

(a) $\sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$

(b) $\sum_{n=0}^{\infty} e^n$

(c) $\sum_{n=1}^{\infty} \frac{1}{n}$.

(d) $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

(e) $\sum_{n=2}^{\infty} \frac{n^2}{n^4-1}$ (Limit Comparison Test)

(f) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2^n}$ (Comparison Test)

(g) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ (Integral Test)