§11.6: Power Series
§11.7: Taylor Series

Name: ______________________

Math 1910

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Rapid Review

(1) An infinite series of the form $F(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$ is called a **power series** and c is called the **center**.

(2) The **radius of convergence** of $F(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$ is a constant R such that $F(x)$ converges absolutely for $|x-c| < R$ and diverges for $|x-c| > R$. If $F(x)$ converges for all x, then $R = \infty$.

(3) To determine R, use ______ (1)

(4) $\sum_{n=0}^{\infty} x^n = \dfrac{1}{1-x}$, with $R =$ ______ (2).

(5) The powerseries $T(x) = \sum_{n=0}^{\infty} \dfrac{f^{(n)}(c)}{n!}(x-c)^n$ is called the **Taylor Series** for $f(x)$. If $c = 0$, this is called a **Maclaurin series**.

(6) $1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} =$ ______ (3)

(7) $(1+x)^a = 1 + \sum_{n=1}^{\infty} \binom{a}{n} x^n$ for $|x| < 1$, where $\binom{a}{n} = \dfrac{a(a-1)(a-2)\cdots(a-n+1)}{n!}$

PROBLEMS

(1) Show that all three of the following power series have the same radius of convergence, but different behavior at the endpoints.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{9^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{n9^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^2 9^n}$

(2) Use the geometric series formula to expand the function $\frac{1}{1+3x}$ in a power series with center $c = 0$ and determine radius of convergence.

(3) Find the Taylor series of the following functions and determine the radius of convergence.

(a) $f(x) = \sin(2x)$, centered at $x = 0$.

(b) $f(x) = e^{4x}$, centered at $x = 0$.

(c) $f(x) = x^2 e^{x^2}$, centered at $x = 0$.

(d) $f(x) = \frac{1}{3x-2}$, centered at $c = -1$.

(e) $f(x) = (1+x)^{1/3}$, centered at $c = 0$.

(f) $f(x) = \sqrt{x}$, centered at $c = 4$.