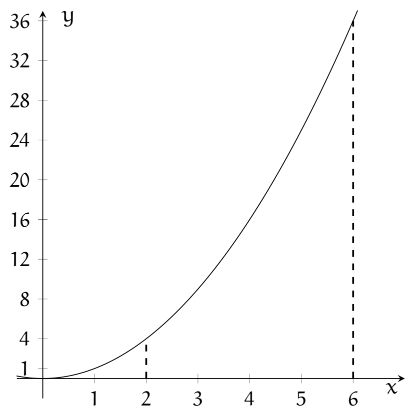
RIEMANN SUMS NAME: ________________

April 26, 2017

Approximating Area Under a Curve



Let's try to approximate the area under the curve $y = x^2$ between $x = 2$ and $x = 6$, as seen in the graph to the left.

We can estimate the area under the curve using rectangles. Let's use four. Since we have four rectangles, the base of each rectangle is 1 unit long, because our interval (from $x = 2$ to $x = 6$) is 4 units long.

So, we have four rectangles with bases in the intervals [2, 3], [3, 4], [4, 5], [5, 6].

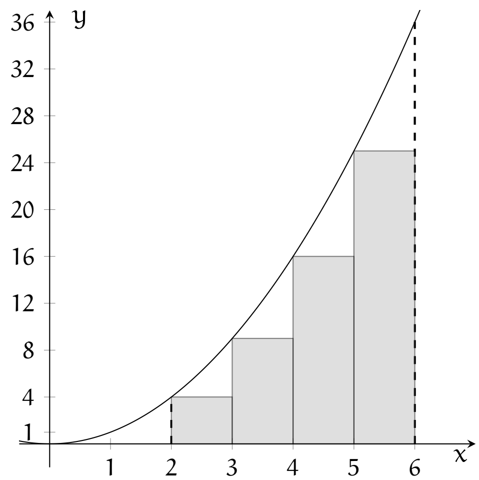
But, where do we get the height of our rectangle from? There are three ways to do this:

(1) Left-handed rectangles

(2) Right-handed rectangles

(3) Midpoint rectangles

Left-Handed Rectangles



When using left-handed rectangles, first evaluate the function at each *left* end point of the intervals.

In our case we have: $f(2) = 4$, $f(3) = 9$, $f(4) = 16$, and $f(5) = 25$.

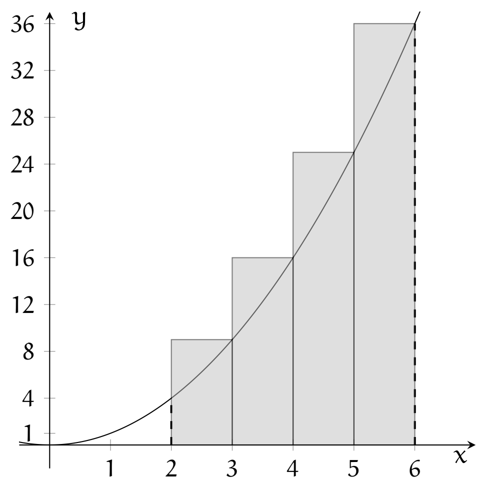
Then, draw a rectangle in each interval with a height equal to that of the value of the function at the left end points, as seen in the graph.

(a) Using the four rectangles what is the approximate area?

(b) Do you think this is an over estimate or an under estimate? Explain.

(c) Write a Riemann sum for this area estimate.

Right-Handed Rectangles



When using right-handed rectangles, first evaluate the function at each *right* end point of the intervals.

In our case we have: $f(3) = 9$, $f(4) = 16$, $f(5) = 25$, and $f(6) = 36$.

Then, draw a rectangle in each interval with a height equal to that of the value of the function at the right end points, as seen in the graph.

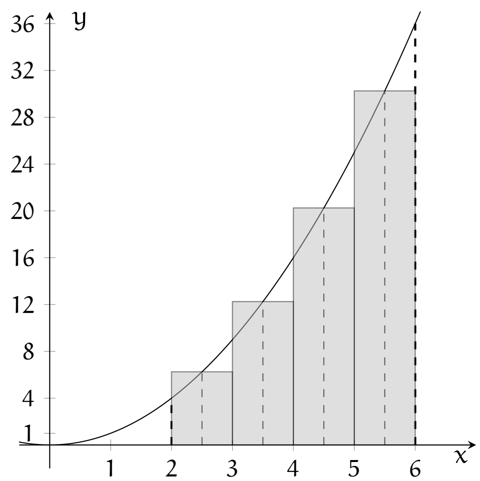
(a) Using the four rectangles what is the approximate area?

(b) Do you think this is an over estimate or an under estimate? Explain.

(c) Write a Riemann sum for this area estimate.

This material is based upon original Active Calculus materials produced by the University of Nebraska at Omaha.

Midpoint Rectangles



When using midpoint rectangles, first find the *midpoint* of each of the intervals.

In our case, the midpoint of [2, 3] is 2.5, the midpoint of [3, 4] is 3.5, the midpoint of [4, 5] is 4.5, and the midpoint of [5, 6] is 5.5.

Then, evaluate the function at each *midpoint* of the intervals. In our case we have $f(2.5) = 6.25$, $f(3.5) = 12.25$, $f(4.5) = 20.25$, and $f(5.5) = 30.25$. Then, draw a rectangle in each interval with a height equal to that of the value of the function at the midpoint, as seen in the graph.

(a) Using the four rectangles what is the approximate area?

(b) Do you think this is an over estimate or an under estimate? Explain.

(c) Write a Riemann sum for this area estimate.

1. Write a Riemann sum to approximate the area with 8 rectangles, using each of the three methods.

   (a) Left-Handed Rectangles

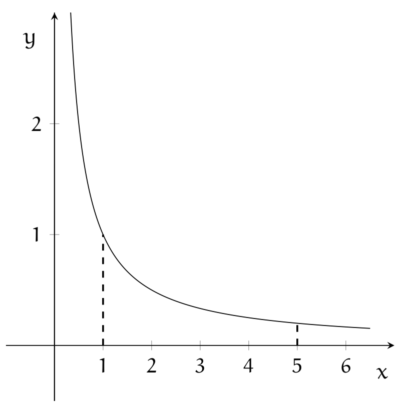
   (b) Right-Handed Rectangles

   (c) Midpoint Rectangles

What would happen if we had 100 rectangles? 1,000 rectangles? An infinite number of rectangles?

This is how we find area under the curve.

Now, you try! Approximate the area under the curve $y = \frac{1}{x}$ from $x = 1$ to $x = 5$ with 4 rectangles, using each of the three methods. The graph is given below.



(a) Write a Riemann sum to estimate this area with 4 rectangles.

(b) Write a Riemann sum to estimate this area with 8 rectangles.

(c) Write a Riemann sum to estimate this area with 1000 rectangles.

(d) Write a Riemann sum to estimate this area with an arbitrary number of rectangles, N.

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