ROLLER COASTER DESIGN PROJECT

Math 1110
Spring 2017

Due March 20, 2017

The goal of this project is to design a rollercoaster and compute it's **thrill**. The roller coaster with the greatest thrill will win a prize! You may work with a partner if you wish.

DEFINITIONS

A **roller coaster** is the graph of a function $r(x)$ with domain $[0, 200]$ such that:

- the roller coaster starts on the ground: $r(0) = 0$.
- the maximum height of the roller coaster is 75 meters: $r(x) \leq 75$ for all $x \in [0, 200]$.
- the roller coaster does not go underground: $r(x) \geq 0$ for all $x \in [0, 200]$.
- the ride is smooth: $r(x)$ is differentiable everywhere on its domain.
- the angle of steepest descent for the roller coaster is never more than 80 degrees ($4\pi/9$ radians).

A **drop** of a roller coaster is an interval on which the graph of the coaster is strictly decreasing.

The **thrill of a drop** is the product of the angle of steepest descent in the drop (in radians) and the total vertical distance of the drop.

The **thrill of a coaster** is the sum of the thrills in each drop. This is the quantity you're trying to maximize.

SUBMISSION

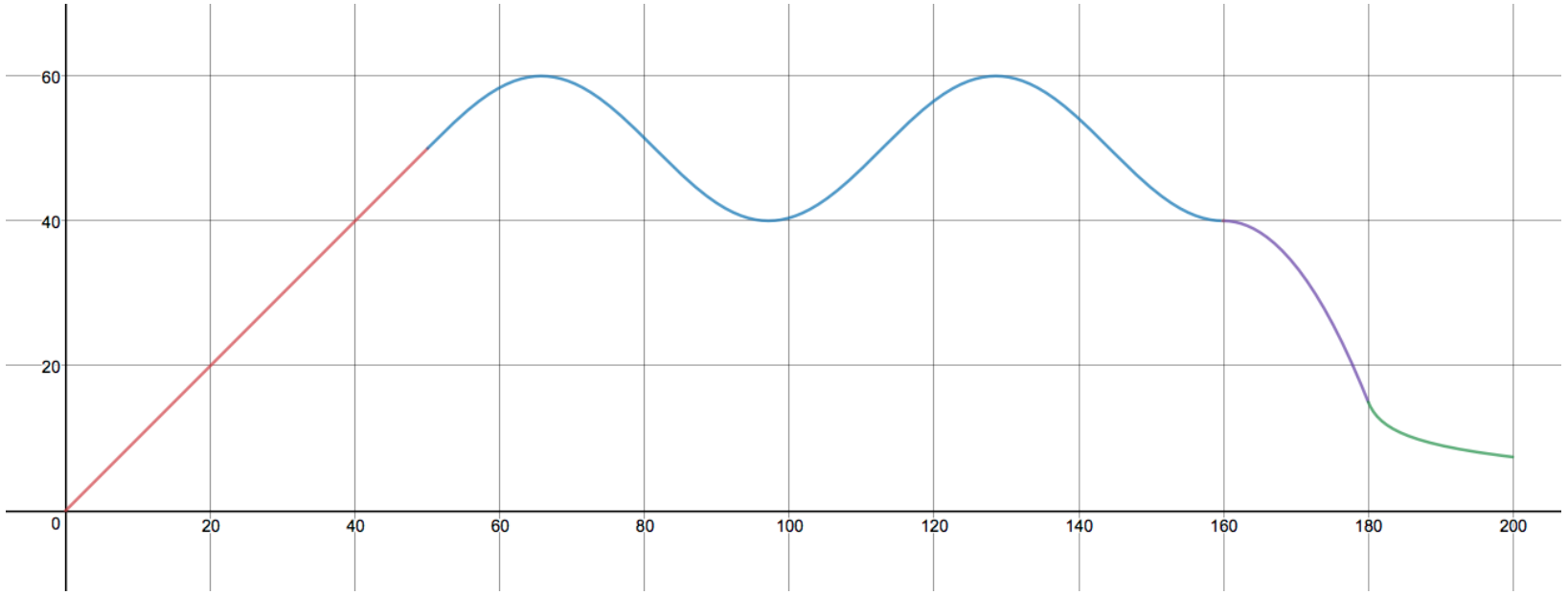
You will submit a report that outlines the design of your roller coaster, including a graph of the coaster and all relevant computations. You will be graded on the following criteria. (50 points total)

MATHEMATICAL CONTENT (35 points)

- The thrill of each drop and the total thrill are computed correctly and clearly stated. (10 points)
- The roller coaster adheres to the constraints of roller coaster design:
  - The graph of the roller coaster passes through the origin, is less than 75 meters tall, is no more than 200 meters long, and does not go underground. (4 points)
  - Calculus is used to demonstrate that the graph of the roller coaster is differentiable everywhere on its domain. (8 points)
  - Calculus is used to find the angle of steepest descent in each drop, and verify that it is less than 80 degrees. (8 points)
- At least three equations for different types of functions (logarithmic, exponential, trigonometric, rational, polynomial, etc.) are used to define your roller coaster. (5 points)

PRESENTATION (15 points)

- The equation for the graph of the roller coaster is clearly listed as a piecewise function. (5 points)
- A graph of the roller coaster is produced using technology (such as desmos.com). (5 points)
- The report is well-organized and easy to follow. Computations are presented neatly by hand or typed. (5 points)



$$y = \begin{cases} x & 0 \le x \le 50 \\ 10\sin\left(\frac{x-50}{10}\right) + 50 & 50 \le x \le 50 + 35\pi \\ 40 & 50 + 35\pi \le x \le 160 \\ -\left(\frac{x-160}{4}\right)^2 + 40 & 160 \le x \le 180 \\ -2.5\ln(x - 179) + 15 & 180 \le x \le 200 \end{cases}$$

A Sample Coaster. There are three drops total, on the intervals $[50 + 5\pi, 50 + 15\pi]$, $[50 + 25\pi, 50 + 35\pi]$, and $[160, 200]$. Each of the first two drops has thrill 20 $(\pi/4)$. The thrill of the final drop is

$$(25 - 2.5\ln(81)) \cdot \arctan(2.5)$$

The total thrill is therefore $40\left(\frac{\pi}{4}\right) + (25 - 2.5\ln(81)) \cdot \arctan(2.5)$, or approximately 51.7